Fuzzy Approach to Estimates Entropy and Risks for Innovative Projects and Programs

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ABSTRACT

The article deals with the development and implementation of innovative development programs as an ordered sequence of decision-making. Innovations are considered as a specific form of chaos or disorder in the transition to a new level of development. As known, innovative projects have a high level of internal and external uncertainties. Therefore they are difficult for evaluated using classical mathematical methods. In this case authors proposed to use fuzzy entropy for the selection of innovative projects. It can successfully act as important mathematically justified criteria permitting controversial selection of the projects. The paper presents definition of fuzzy entropy and corresponding formulation of the problem. Numerical example of fuzzy entropy calculation under conditions of uncertainty has been given. Then also has been developed fuzzy approach for evaluating the risk of innovative projects in the conditions of uncertainty. Presents the general formulation of the problem, describes the ways of its solutions, presented graphical interpretation and analyzed the results. Thus, for evaluating risks of investment projects can be used the apparatus of fuzzy sets which takes into account the non-recurring conditions and a high level of uncertainty in their implementation. Considered approach allows select projects eligible for inclusion in innovative development program. This selection taking into account the regional innovation potential, the presence of uncertainties with the different nature of the occurrence.

INTRODUCTION

As is well known, multi-step process of development and implementation of innovative development programs is a logically ordered sequence of decision-making. It focuses not only on the formation of efficient portfolios of innovative projects, but also to establish an acceptable
level of risk for regional innovation programs based on both internal and external uncertainties influencing its effectiveness and feasibility.

Like all open systems, economic systems and their elements constantly and dynamically change depending on external or internal factors. These changes result from chaos to order, and vice versa. At that innovations can be seen as a specific form of chaos or disorder in the transition to the next level of development. Therefore, in this case, the activities of the relevant authorities, which is aimed in some way to regulate innovation can be defined as a form of "control" of chaos or disorder (Mokiy et al, 2012; Rus, 2012; Trifonova, 2012; Duin and Hermeler, 2014; Janssen et al, 2015).

1. INFORMATION FEATURES OF INNOVATIVE PROJECTS

It’s obvious that the process of formation of innovative programs of regional development requires a huge amount of background information to make responsible decisions (Radosevic, 2002; Amorós and López, 2011; Martin et al, 2011; Wamser et al, 2013; Rinkinen, 2015). It is possible to use multi-criteria decision making tools for selection of alternatives based on various important, sometimes conflicting criteria (Bauers, Zavadskas, 2010; Streimikiene, 2013; Streimikiene, Balezentiene, 2012; Streimikiene et al., 2011; Kaplikski, Tupenaite, 2010; Zvirblis A, Buracas, 2012).

Therefore it is appropriate mention of the law of requisite variety formulated by Ashby, according to which only diversity can "destroy" the diversity, and the entropy is seen as a characteristic of diversity (Ashby, 1981).

This is especially important in the implementation of innovations and projects in general, due to the high degree of risk when making decisions under conditions of uncertainty. Because each innovation project in varying degrees, is unique, and the possibility in the new environment to successfully use the accumulated experience of previous decisions is restricted.

Governing bodies who lead the selection of innovative projects and the formation of innovative development programs, to reduce the risk of making wrong decisions, in the selection process should include experts. They are in this case "the source of the information" for that type of innovative projects. Thus the decrease in entropy (uncertainty), which is defined as an objective measure of places without information about the phenomenon, object or system should be based on expert assessments. However, this is only possible with a high information content of not only individual experts, but also the whole of the expert group.

At the same time because the innovation projects have a high level of uncertainty, they are very difficult to estimate, using classical mathematical methods. Thus, the most important features of innovation projects is imperfect information and its vagueness, which measure can serve the fuzzy entropy (Leh and Wang, 2001; Tang and Leung, 2009; Fasanghari et al, 2011; Wei and Tang, 2011; Cavallaro et al, 2016). It is noteworthy that the authors over the years developed similar to proposed approaches to modeling of various economic problems on the basis of the theory of fuzzy sets and fuzzy modeling (Chernov et al, 2010; 2015; 2016)

2. FUZZY ENTROPY AS A MEASURE OF UNCERTAINTY

It is well known that in some sources the entropy of as the concept of the theory of probability is defined as "not a very good indicator". Reason for this is that it is not directly dependent on the absolute values of the membership functions, but from their relative values. However, entropy as a transdisciplinary concept nevertheless has more wide applications, including in the theory of fuzzy sets. Moreover, under specific conditions, in particular, the selection of innovative projects, it may be an important criterion for the choice of resolution for disputable projects. As a measure of uncertainty entropy could lead to two effects that characterize the chances and
risks. In the presence of entropy in the innovation system the region has a chance to win and (at the same time) there is a risk to lose.

Suppose that during the formation of an innovative program examines a variety of innovative projects \( P = \{ P_i : i = \overline{1, I} \} \) pretending to be included in innovation program \( V_k \).

After all the projects (to which fuzzy evaluation of experts on multitude of criteria \( C = \{ C_j : j = \overline{1, J} \} \) must be assigned) with a variety of linguistic evaluations

\[ L_C = \left\{ \left. \left( \mu_{C_j}(P_i), b \right) \right| j = \overline{1, J}, k = \overline{1, K} \right\} \]

will be selected, will be chosen desired set of projects

\[ P^+ = \{ P_i^+ : l = \overline{1, L}, i \} \]

where \( l < i, P^+ \subset P \) that is to say \( C : P_i \rightarrow P_i^+ \in P^+ \subset P \).

Then the risk of erroneous evaluations of innovative projects quantitatively can be estimated through the uncertainty of linguistic variables \( C \). The level of evaluation \( P_i \in P, i = \overline{1, I} \) by the criterion \( C_j : j = \overline{1, J} \) is characterized by a number \( \mu_{C_j}(P_i) \in [0,1] \). Each value of the evaluation for criteria \( C_j \) includes not only information about the project \( P_i \), but also the uncertainty \( (H) \) that is \( H(\mu_{C_j}(P_i)) + I(\mu_{C_j}(P_i)) = 1 \).

And in evaluation \( \mu_{C_j}(P_i) \) the value of entropy depends on expert awareness. As is well known in the theory of information the entropy (uncertainty) \( H(P_i) \) determined by the Shannon formula. If taking into account fuzziness of evaluations, it can be represented as follows:

\[ H(P_i) = \sum_{j=1}^{J} \mu_{C_j}(P_i) \log_2 \mu_{C_j}(P_i) \]

It is obvious that \( H_{\text{min}} = 0 \) when \( \mu_{C_j}(P_i) = 1, j = \overline{1, J}, i = \overline{1, I} \) or \( \mu_{C_j}(P_i) = 0, j = \overline{1, J}, i = \overline{1, I} \), that is, when the project is indisputably included in the program or is rejected, \( H_{\text{max}} = \log_2 J \), at \( \mu_{C_1}(P_1) = \mu_{C_2}(P_2) = \ldots = \mu_{C_J}(P_i) = 1 \), that is, when all projects have the same chances to enabling or disabling.

If the inequality \( H(P_i^+) \geq H(P_i) \) is valid for the entropy of the selected projects it means that an erroneous decision was made. And conversely, if \( H(P_i^+) < H(P_i) \) then the decision is correct. Then we can assert that \( H(P_i^+) = H(P_i) - I(C_j | P_i) \), \( I(C_j | P_i) = H(C_j) - H(C_j | P_i) \) shows the amount of information in the criteria \( C_j \) for projects \( P_i \), and \( H(C_j | P_i) \) is the conditional uncertainty. Hence, the selection of projects in the main depends from \( H(C_j) \), because \( H(C_j | P_i) \to 0 \) when correct choosing.

In the literature fuzzy entropy is also determined as follows:
\[ H(P_i) = \frac{\sum_{C \subseteq \mathcal{C}} \text{count}(C \cap \overline{C})}{\sum_{C \subseteq \mathcal{C}} \text{count}(C \cup \overline{C})} = \frac{\sum_{j=1}^{J} (\mu_{C_j}(P_i) \cap \mu_{C_j}(P_i))}{\sum_{j=1}^{J} (\mu_{C_j}(P_i) \cup \mu_{C_j}(P_i))} \]

where \( \mu_{C_j}(P_i) \) — are the supplements, and \( \sum \text{count}(C) = \sum \mu_{C_j}(P_i) \).

If for clear sets \( C \cap \overline{C} = \emptyset \), then for fuzzy sets this intersection in general is not always empty and is in the range \([0,1]\). So, guided by all the above provisions and using entropy evaluation we can determine a more effective set of projects for inclusion in the program.

### 3. Numerical Example of Fuzzy Entropy Calculation

Let suppose that during the formation of an innovative program there are examines a variety of innovative projects \( P = \{P_i : i = 1,5\} \) eligible for inclusion in the innovative program \( V_k \).

And after the evaluation only three projects can be included in the program. At the beginning stage, all projects chance of getting into the program are equal. Obviously, in this case, the entropy is maximum and the Shannon measure coincides with the Hartley measure \( H(P_{i=1-3}) = \log_5 5 = 1 \) and \( \sum_{i=1}^{5} H(P_{i=1-3}) = 5 \) is available for the program.

At the next stage to reduce the entropy in the analysis process we will involve experts who based on specified criteria \( C_j, j = 1,3 \) assess the feasibility of the level of inclusion projects in the program. A simplified variant of such estimates is presented in Table 1.

**Table 1. Criteria estimates for projects**

<table>
<thead>
<tr>
<th>Criteria 1</th>
<th>Criteria 2</th>
<th>Criteria 3</th>
<th>MaxMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project 1</td>
<td>1</td>
<td>0.95</td>
<td>0.9</td>
</tr>
<tr>
<td>Project 2</td>
<td>0.8</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Project 3</td>
<td>0.85</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>Project 4</td>
<td>0.6</td>
<td>0.8</td>
<td>0.95</td>
</tr>
<tr>
<td>Project 5</td>
<td>1</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Source: authors’ formulation of the problem (numerical example)

Usually in such conditions is used (as the most simple) maximin method for ranking of projects. According to this method from five considered projects indisputable will be selected the first, and the second after him. However, the third in the solution may be either the second or the fifth project. Of course, other criteria also may be used (such as the average score criterion, Savage criterion). The common disadvantage of these methods is that they do not provide a reliable ranking in the case of similar characteristics of alternatives. Therefore joint use of these criteria is recommended to improve the reliability of results. But quite often in this case are obtained not coinciding estimates as they themselves (these criteria) are based on initial contradictory hypotheses. In contrast, the entropy approach (Shannon entropy, fuzzy entropy) is free from these contradictions.

Since the expert estimates by their nature are unclear, so the estimates presented in the
Table 1 can be considered as the value of membership functions characterizing the degree of the possibility of including projects in the innovative program. Then it is possible to calculate the entropy of Shannon as:

\[
H(P_1) = 1 \log_2[1]^{-1} + 0.95 \log_2[0.95]^{-1} + 0.9 \log_2[0.9]^{-1} \approx 0 + 0.038 + 0.09 \approx 0.13;
\]

\[
H(P_2) = 0.8 \log_2[0.8]^{-1} + 0.95 \log_2[0.95]^{-1} + 0.95 \log_2[0.95]^{-1} \approx 0.16 + 0.038 + 0.038 \approx 0.24;
\]

\[
H(P_3) = 0.85 \log_2[0.85]^{-1} + 1 \log_2[1]^{-1} + 0.9 \log_2[0.9]^{-1} \approx 0.13 + 0 + 0.09 \approx 0.22;
\]

\[
H(P_4) = 0.6 \log_2[0.6]^{-1} + 0.8 \log_2[0.8]^{-1} + 0.95 \log_2[0.95]^{-1} \approx 0.28 + 0.16 + 0.038 \approx 0.48;
\]

\[
H(P_5) = 1 \log_2[1]^{-1} + 0.8 \log_2[0.8]^{-1} + 0.8 \log_2[0.8]^{-1} \approx 0 + 0.16 + 0.16 \approx 0.32.
\]

Since the entropy describes level of uncertainty, the best project should have the lowest entropy. Accordingly ranking of projects is as follows: \(H(P_1) = 0.13\); \(H(P_2) = 0.22\); \(H(P_3) = 0.24\); \(H(P_4) = 0.32\); \(H(P_5) = 0.48\).

This confirms the previous (MaxMin approach) result but in this case estimates for all projects has a large degree of difference and unambiguously resolved the situation with the second and the fifth project. After the evaluation of innovative projects by experts the volume of information in the criteria makes accordingly:

\[
I(C \mid P_1) = 1 - H(P_1) = 0.87; \quad I(C \mid P_2) = 0.76; \quad I(C \mid P_3) = 0.78; \quad I(C \mid P_4) = 0.52; \quad I(C \mid P_5) = 0.68.
\]

If we intend to consideration of several innovative programs, they can also compare the value of entropy for the program, which is determined as \(\sum_{i=1}^{5} H(P_{i,i=1,5})\) and in our case is equal to 1.39.

As has been discussed above, when ranking of alternatives in uncertainty we can use several variants for valuation. Among others we can (for example) use another alternative relation also given above and obtain these numerical results:

\[
H(P_1) = \frac{(1 \cap 0) + (0.95 \cap 0.05) + (0.9 \cap 0.1)}{(1 \cup 0) + (0.95 \cup 0.05) + (0.9 \cup 0.1)} = \frac{0 + 0.05 + 0.1}{1 + 0.95 + 0.9} = \frac{0.15}{2.85} = 0.05;
\]

\[
H(P_2) = \frac{(0.8 \cap 0.2) + (0.95 \cap 0.05) + (0.95 \cap 0.05) + (0.8 \cap 0.02) + (0.95 \cup 0.05) + (0.95 \cup 0.05) + (0.8 \cap 0.02) + (0.95 \cup 0.05) + (0.95 \cup 0.05)}{0.8 + 0.95 + 0.95} = \frac{0.2 + 0.05 + 0.05}{2.7} = 0.11;
\]

\[
H(P_3) = \frac{(0.85 \cap 0.15) + (1 \cap 0) + (0.9 \cap 0.1)}{(0.85 \cup 0.15) + (1 \cup 0) + (0.9 \cup 0.1)} = \frac{0.15 + 0 + 0.1}{0.85 + 1 + 0.9} = \frac{0.25}{2.75} = 0.09;
\]

\[
H(P_4) = \frac{(0.6 \cap 0.4) + (0.8 \cap 0.2) + (0.95 \cap 0.05) + (0.6 \cap 0.4) + (0.8 \cap 0.2) + (0.95 \cap 0.05)}{0.6 + 0.8 + 0.95} = \frac{0.4 + 0.2 + 0.05}{2.35} = 0.28;
\]

\[
H(P_5) = \frac{(1 \cap 0) + (0.8 \cap 0.2) + (0.8 \cap 0.2) + (0.8 \cap 0.2) + (0.8 \cap 0.2) + (1 \cup 0) + (0.8 \cap 0.2) + (0.8 \cap 0.2)}{1 + 0.8 + 0.8} = \frac{0.4}{2.6} = 0.15.
\]

Then preferability of projects on the second method is as follows: \(P_6 \neq P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5\), therefore we get the same result, but at various calculation methods. Thus we can unambiguously and confidently assert that during the formation of innovative programs of alternative projects, we can use entropy (as a selection criterion) calculated on the proposed two variants of relations.

In case if the criteria in different orders (ranks) have similar results (that is the entropies are equal), we can estimate the entropy of criteria taking into account their degree of importance.
4. FUZZY FACTORS FOR RISKS IN INNOVATIVE PROJECTS

In the economic sphere, where there is a clearly established budget and limited resources, it becomes increasingly important to question risk assessment and management.

It is obvious that innovation, like any other business process, is always accompanied by uncertainty. But unlike other economic activities, it (as such) has a more high level of uncertainty and, accordingly, an increased risk. Generally, innovative program development of the region can be seen as the result of an open, dynamic, innovative social cooperation of innovative participants at various levels. From this it follows that every innovative project (which is part of this program) bears the internal and external factors of uncertainty that define the character of development and the effectiveness of the innovation program.

Accordingly, when the region chooses a course of innovative development, its objective should be "golden mean" between uncertainty and its handling. It is also clear that such activity is not accurate predestination, but only a fuzzy system of desired results. Thus, when deciding on the formation and implementation of innovative program development in the region must be objectively and reasonably assess the effectiveness of the riskiness of and innovation projects.

The risk assessment task of innovative development programs is associated with several difficulties due to the following factors. The absence of a unified database to theoretically assess the risks of innovative programs and their components which is due to the uniqueness of this sphere of activity. The plurality of components of uncertainty may be interrelated because the influence of these factors has dynamic and non-linear character.

Consequently, the assessment of the probability of the risk and its consequences can’t be obtained by statistical methods. This allows considering the reasonable used for this task the fuzzy sets methodology.

5. PROBLEM FORMULATION AND SOLUTION FOR FUZZY RISK

Suppose there is an innovative program \( V \) in the framework of which addresses the several innovative projects \( P(V) = \{ P_k : k = 1, K \} \) pretending to inclusion in the investment program.

Assume there is a vector of innovative projects risks \( R = \{ R_i : i = 1, I \} \) and for each \( R_i \) can be allocated the set of structural factors of uncertainty \( R_i = \{ r_{ij} : i = 1, I, j = 1, J \} \), where \( I \neq J \).

And that for each structural index we can specify the class of linguistic estimates \( L_{R_i} = \{ t_{ij} \} \), term-sets \( T = \{ T_i \} \) and the fuzzy set \( M_{R_i} = \mu(t_{ij}) \). It is obvious that projects (that make innovative programs) may belong to different economic spheres. Hence, their risks and factors of uncertainty can have a different nature of influence.

Accordingly, it is necessary to take into account the weights of risks and structural indices (factors) \( W = \{ W_i : i = 1, I \} \) and \( W_i = \{ w_{ij} \} \). If the estimation of the structural indexes \( t_{ij} \) on the sign \( t \) from the side of \( n \)-th expert we mark through \( q_{ij}^{(n)} : i = 1, I, j = 1, J, n = 1, N, t = 1, T \), then generalized estimation taking into account the weight coefficients will be as follows

\[
\xi_{ij} = \sum w_{ij} \times q_{ij}^{(n)} / n
\]

and thus can be formed fuzzy evaluation matrix \( \xi_{ij} \):
Further risk assessment can be obtained using two different variants.

First, \( R_i^+ = \min_{i=1}^{I} \xi_{ijt} \), when the occurrence of the risk is unlikely.

Second, \( R_i^- = \max_{i=1}^{I} \xi_{ijt} \), when probability of risk appearance is high,

where \( T^+ \) and \( T^- \) correspondingly are low and high linguistic evaluations.

Because for innovation activity uncertainty can mean both risks and the chances \( S \), for the innovative project \( S_p = R_p^{+(-)} = 1 - R_p^{-(-)} \). Then the integral risk of the innovation program may be defined as: \( R_r^+ = \sum_{k=1}^{K} R_p^+ \) or \( R_r^- = \sum_{k=1}^{K} R_p^- \).

### 6. NUMERICAL EXAMPLE OF FUZZY RISK CALCULATION

Suppose that 6 classes of risks were identified for innovative programs with \( R = \{ R_i, i = 1, 6 \} \). Structural indexes for each (Table 2) are \( R_1 = \{ r_{ij}, j = 1,3 \} \), \( R_2 = \{ r_{ij}, j = 4 \} \), \( R_3 = \{ r_{ij}, j = 1,2 \} \), \( R_4 = \{ r_{ij}, j = 1,3 \} \), \( R_5 = \{ r_{ij}, j = 1,3 \} \) and they have weight coefficients \( W_i = \{ w_{ij} \} \in [0,1] \).

<table>
<thead>
<tr>
<th>Class index(risk of)</th>
<th>Description</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>timing risks ( R_1 )</td>
<td>Optimistic execution plan</td>
<td>( R_{11} )</td>
</tr>
<tr>
<td></td>
<td>Weak control of the execution time</td>
<td>( R_{12} )</td>
</tr>
<tr>
<td></td>
<td>The lack of a reserve of time</td>
<td>( R_{13} )</td>
</tr>
<tr>
<td>risks of human resources ( R_2 )</td>
<td>The rationality of structure</td>
<td>( R_{21} )</td>
</tr>
<tr>
<td></td>
<td>The level of qualification</td>
<td>( R_{22} )</td>
</tr>
<tr>
<td></td>
<td>Flowability and motivation of personnel</td>
<td>( R_{23} )</td>
</tr>
<tr>
<td></td>
<td>The experience of leaders</td>
<td>( R_{24} )</td>
</tr>
<tr>
<td>risks of material resources ( R_3 )</td>
<td>Level of the material base</td>
<td>( R_{31} )</td>
</tr>
</tbody>
</table>
Table 3. Numerical fuzzy risk assessment

<table>
<thead>
<tr>
<th>Linguistic risk assessments</th>
<th>The corresponding fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>(0.1 0.2 0.3)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.25 0.35 0.5)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.4 0.5 0.6)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.55 0.65 0.75)</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>(0.7 0.8 0.9)</td>
</tr>
<tr>
<td>Critical (EX)</td>
<td>(0.85 0.9 1)</td>
</tr>
</tbody>
</table>

Source: own authors’ numerical example

Figure 1. Fuzzy membership functions for linguistic risk assessments

Source: own authors’ numerical example

It should be noted that the triangular membership functions in this case are chosen only from reasons of simplicity for the subsequent calculations. Numerical fuzzy evaluation of risks for innovative projects is presented in the Table 4.
Table 4. Numerical fuzzy evaluation of risks for innovative projects

<table>
<thead>
<tr>
<th>Index</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>0.1 0.2 0.3</td>
<td>0.4 0.5 0.6</td>
<td>0.7 0.8 0.9</td>
<td>0.1 0.2 0.3</td>
</tr>
<tr>
<td>R_11</td>
<td>0.1 0.2 0.3</td>
<td>0.4 0.5 0.6</td>
<td>0.7 0.8 0.9</td>
<td>0.1 0.2 0.3</td>
</tr>
<tr>
<td>R_12</td>
<td>0.5 0.4 0.5</td>
<td>0.85 0.9 1</td>
<td>0.7 0.8 0.9</td>
<td>0.6 0.25 0.35</td>
</tr>
<tr>
<td>R_13</td>
<td>0.9 0.1 0.2</td>
<td>0.25 0.35 0.5</td>
<td>0.4 0.5 0.6</td>
<td>0.7 0.25 0.35</td>
</tr>
<tr>
<td>R_2</td>
<td>0.7 0.25 0.35</td>
<td>0.25 0.85 0.9</td>
<td>0.7 0.8 0.9</td>
<td>0.6 0.25 0.35</td>
</tr>
<tr>
<td>R_21</td>
<td>0.1 0.2 0.3</td>
<td>0.1 0.2 0.3</td>
<td>0.7 0.8 0.9</td>
<td>0.6 0.4 0.5</td>
</tr>
<tr>
<td>R_22</td>
<td>0.4 0.5 0.6</td>
<td>0.7 0.8 0.9</td>
<td>0.85 0.9 1</td>
<td>0.7 0.25 0.35</td>
</tr>
<tr>
<td>R_23</td>
<td>0.3 0.7 0.8</td>
<td>0.7 0.8 0.9</td>
<td>0.3 0.5 0.6</td>
<td>0.6 0.25 0.35</td>
</tr>
<tr>
<td>R_24</td>
<td>0.3 0.7 0.8</td>
<td>0.7 0.8 0.9</td>
<td>0.3 0.5 0.6</td>
<td>0.6 0.25 0.35</td>
</tr>
<tr>
<td>R_3</td>
<td>0.3 0.7 0.8</td>
<td>0.25 0.35 0.5</td>
<td>0.7 0.8 0.9</td>
<td>0.34 0.55 0.65</td>
</tr>
<tr>
<td>R_31</td>
<td>0.3 0.7 0.8</td>
<td>0.25 0.35 0.5</td>
<td>0.7 0.8 0.9</td>
<td>0.34 0.55 0.65</td>
</tr>
<tr>
<td>R_32</td>
<td>0.4 0.5 0.6</td>
<td>0.4 0.5 0.6</td>
<td>0.85 0.9 1</td>
<td>0.35 0.7 0.8</td>
</tr>
<tr>
<td>R_4</td>
<td>0.7 0.25 0.35</td>
<td>0.25 0.35 0.5</td>
<td>0.7 0.25 0.35</td>
<td>0.34 0.55 0.65</td>
</tr>
<tr>
<td>R_41</td>
<td>0.1 0.2 0.3</td>
<td>0.7 0.8 0.9</td>
<td>0.4 0.5 0.6</td>
<td>0.34 0.55 0.65</td>
</tr>
<tr>
<td>R_5</td>
<td>0.4 0.5 0.6</td>
<td>0.25 0.35 0.5</td>
<td>0.55 0.4 0.5</td>
<td>0.3 0.7 0.8</td>
</tr>
<tr>
<td>R_51</td>
<td>0.4 0.5 0.6</td>
<td>0.25 0.35 0.5</td>
<td>0.55 0.4 0.5</td>
<td>0.3 0.7 0.8</td>
</tr>
<tr>
<td>R_52</td>
<td>0.5 0.4 0.5</td>
<td>0.55 0.65 0.75</td>
<td>0.55 0.4 0.5</td>
<td>0.3 0.7 0.8</td>
</tr>
<tr>
<td>R_6</td>
<td>0.25 0.85 0.9</td>
<td>0.55 0.65 0.75</td>
<td>0.55 0.4 0.5</td>
<td>0.25 0.85 0.9</td>
</tr>
<tr>
<td>R_61</td>
<td>0.25 0.85 0.9</td>
<td>0.55 0.65 0.75</td>
<td>0.55 0.4 0.5</td>
<td>0.25 0.85 0.9</td>
</tr>
<tr>
<td>R_62</td>
<td>0.3 0.7 0.8</td>
<td>0.25 0.35 0.5</td>
<td>0.55 0.65 0.75</td>
<td>0.3 0.7 0.8</td>
</tr>
<tr>
<td>R_63</td>
<td>0.4 0.5 0.6</td>
<td>0.1 0.2 0.3</td>
<td>0.6 0.4 0.5</td>
<td>0.3 0.7 0.8</td>
</tr>
</tbody>
</table>

Source: own authors’ numerical example

Considering weighting factors for each project we obtain the results shown in the Table 5.

Table 5. Numerical fuzzy evaluation in view of weight coefficients

<table>
<thead>
<tr>
<th>Index</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>0.1 0.2 0.3</td>
<td>0.16 0.20 0.24</td>
<td>0.21 0.24 0.27</td>
<td>0.1 0.2 0.3</td>
</tr>
<tr>
<td>R_11</td>
<td>0.1 0.2 0.3</td>
<td>0.16 0.20 0.24</td>
<td>0.21 0.24 0.27</td>
<td>0.1 0.2 0.3</td>
</tr>
<tr>
<td>R_12</td>
<td>0.2 0.25 0.3</td>
<td>0.21 0.24 0.27</td>
<td>0.21 0.24 0.27</td>
<td>0.15 0.21 0.3</td>
</tr>
<tr>
<td>R_13</td>
<td>0.9 0.18 0.27</td>
<td>0.125 0.175 0.25</td>
<td>0.24 0.3 0.32</td>
<td>0.175 0.245 0.35</td>
</tr>
<tr>
<td>R_2</td>
<td>0.175 0.245 0.35</td>
<td>0.21 0.24 0.27</td>
<td>0.21 0.24 0.27</td>
<td>0.15 0.21 0.3</td>
</tr>
<tr>
<td>R_21</td>
<td>0.1 0.2 0.3</td>
<td>0.1 0.2 0.3</td>
<td>0.21 0.24 0.27</td>
<td>0.24 0.3 0.32</td>
</tr>
<tr>
<td>R_22</td>
<td>0.22 0.26 0.3</td>
<td>0.175 0.21 0.24</td>
<td>0.21 0.225 0.25</td>
<td>0.175 0.245 0.35</td>
</tr>
<tr>
<td>R_23</td>
<td>0.21 0.24 0.27</td>
<td>0.21 0.24 0.27</td>
<td>0.22 0.26 0.3</td>
<td>0.15 0.21 0.3</td>
</tr>
<tr>
<td>R_24</td>
<td>0.21 0.24 0.27</td>
<td>0.21 0.24 0.27</td>
<td>0.22 0.26 0.3</td>
<td>0.15 0.21 0.3</td>
</tr>
<tr>
<td>R_3</td>
<td>0.21 0.24 0.27</td>
<td>0.15 0.21 0.3</td>
<td>0.175 0.21 0.24</td>
<td>0.187 0.221 0.255</td>
</tr>
<tr>
<td>R_31</td>
<td>0.21 0.24 0.27</td>
<td>0.15 0.21 0.3</td>
<td>0.175 0.21 0.24</td>
<td>0.187 0.221 0.255</td>
</tr>
<tr>
<td>R_32</td>
<td>0.22 0.26 0.3</td>
<td>0.24 0.3 0.32</td>
<td>0.23 0.243 0.27</td>
<td>0.245 0.28 0.315</td>
</tr>
<tr>
<td>R_4</td>
<td>0.175 0.245 0.35</td>
<td>0.2 0.28 0.4</td>
<td>0.175 0.245 0.35</td>
<td>0.187 0.221 0.255</td>
</tr>
<tr>
<td>R_41</td>
<td>0.175 0.245 0.35</td>
<td>0.2 0.28 0.4</td>
<td>0.175 0.245 0.35</td>
<td>0.187 0.221 0.255</td>
</tr>
</tbody>
</table>
A graphical representation of these results is presented in the Figures 2 – 7. All calculations and plot assay are done in Fuzicalc special program for fuzzy operations.

**Figure 2.** Overall risk: favorable variant (0.22, 0.23, 0.24), adverse variant (0.09, 0.2, 0.35)

Source: own authors’ numerical calculations

**Figure 3.** Overall risk: favorable variant (0.24), adverse variant (0.1, 0.21, 0.32)

Source: own authors’ numerical calculations
Figure 4. Overall risk: favorable variant (0.24), adverse variant (0.16, 0.24, 0.35)

Source: own authors’ numerical calculations

Figure 5. Overall risk: favorable variant (0.24, 0.245, 0.25), adverse variant (0.1, 0.22, 0.35)

Source: own authors’ numerical calculations

Figure 6. Overall risk: favorable variant (0.24)

Source: own authors’ numerical calculations
Figure 7. Overall risk: adverse variant (0.09 0.21 0.35)

Source: own authors’ numerical calculations

CONCLUSION

Because for innovation activity uncertainty can mean both the risks and the chances $S$, then for an innovation project $S_P = R_P^{+(-)} = 1 - R_P^{+(-)}$.

For the project $P_2$, when the occurrence of the risk is unlikely, the chances are equal $S_{P_2} = R_{P_2}^{+} = 1 - 0.24 = 0.76$. Then the integral risk of the innovation program may be defined as $R_{P_2}^{+} = \bigcup_{k=1}^{4} R_{P_k}^{+} = (0.09 \ 0.21 \ 0.35)$.

Thus, when evaluating the integral risk of investment projects can be used the apparatus of fuzzy sets, because it takes into account the non-recurring conditions and a high level of uncertainty in their implementation.

As final of this article let us note that it considered the model which allows to select projects eligible for inclusion in the program of innovative development of the region. This selection is carried out taking into account the regional innovation potential, as well as the presence of uncertainties with the different nature of the occurrence.

REFERENCES


