



## Risk Management in Energy Sector Using Short Call Ladder Strategy

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### ARTICLE INFO

Received May 02, 2016  
Received in revised from June 22, 2016  
Accepted August 18, 2016  
Available online 15 September 2016

**JEL classification:**

G11, G13, G32

**DOI:** 10.14254/1800-5845.2016/12-3/3

**Keywords:**

option strategy,  
hedging,  
vanilla option,  
barrier option

### ABSTRACT

The purpose of the paper is to focus on selected aspects of the hedging using of Short Call Ladder strategy created by barrier options. Given barrier option strategies belong to the appropriate tools widely used for risk management with an effective solution for limiting the loss from underlying asset's price development. There is investigated the motivations and practice of firms with regard to using options in their risk management activities. Methodology of the paper is based on European up and knock-in call options together with standard call and barrier call options in analytical expression, which are used for investigation of hedging strategies in increasing markets. According to Haug model, barrier option prices are calculated, due to lack of real data in the market. Based on theoretical models of suitable hedged profit functions in analytical expressions, there are analysed their benefits and risks point of view in general. The requirements of zero-cost option strategy have to be met. Our theoretical results are applied to the Energy Select Sector SPDR ETF, where designed hedged portfolios are analysed and compared to each other with the recommendations for investors. According to our findings we can recommend the hedging variant 2A or 4 as the best variant ensuring the lowest costs at expected intervals of the shares spot price at the maturity date. However, the selection of the right hedged portfolio is based on suitable combinations of the strike prices, the lower and the upper barriers for the best hedging profit function's achievement.

### INTRODUCTION

Two major global crises characterized recent years have produced, among other effects, a dramatic soaring of volatility. In these circumstances, it is fundamental for firms or business leaders to have effective hedging strategies, in order to set up so as to avoid the disruptive consequences of price increase. The methods and mainly instruments used to manage the market

risk are continuously developing. Mainly financial derivatives are used to protect against market risks. Due to liquidity, cost effectiveness and flexibility, options and option strategies are mainly used in the risk management.

Over the last years, the risk management has attracted the attention of a large body of financial studies, which has investigated both the theory and the practice of firm hedging using derivatives. Many empirical papers as Bajo et al. (2014), Brown (2001) and Guay and Kothari (2003) have investigated hedging portfolio strategies using financial derivatives. Ahn et al. (1999) and Loss (2012) studies optimal hedging strategies using options and Hankins (2011) investigates how the firms manage a risk by examining the interactions between financial and operational hedging. Annaert et al. (2007) deal with a bond portfolio using put options in risk management. Recently years, numerous studies have investigated the risk management by using of option strategies, mainly classic vanilla options. To our best knowledge, our risk management using barrier options appears to be unique in this field. Hence, comparing the various risk management variants using Short Call Ladder strategy created by barrier options fills a noticeable gap in this topic. The theoretical results of our analysis will be useful not only for financial institutions, but also for academic and research community.

Option strategies based on barrier options are significant part of our approach used on hedging. Options are conditional contracts representing agreements between 2 parties. The buyers (option holder) and the sellers (option writer) do not have the equal rights and obligations. The option holder has the right to buy (a purchase call) or to sell (a purchase put) and the option writer has the obligation to sell (a sale call) or to buy (a sale put) an underlying asset at a pre-determined time (denoted as the expiration date or the maturity date) at a pre-determined price (denoted as the strike price or the exercise price). For this right, the option premium (or option price) is paid by the option holder to the option writer. The option expiration can be either at the maturity date (European style) or at any time within a pre-determined option expiration period (American style). Options strategies are presented in the studies of Hull (2012) and Kolb (1995). Barrier options are modified from classic vanilla options with the condition of the barrier level. The barrier level can be specified UP or DOWN and its reaching or not reaching before expiration period causing the possibility of option's activation or deactivation (IN or OUT). We know 4 types of the barrier options, i.e. up and knock-in (UI), up and knock-out (UO), down and knock-in (DI), down and knock-out (DO), all for both call and put options. Taleb (1997) and Zhang (1998) deal more detailed characteristics of barrier options.

In general, options provide as well-suited risk management tools to shape different payoff structures to suit various market conditions and to satisfy plentiful requirements of different investors. There are total 124 possible hedging combinations created by Short Call Ladder strategy using barrier options. We have excluded those, which only partially securing the buying price. Our findings show, that for hedging against price increase, the best variants are those created by UI call options with the highest strike price either with combination of the standard vanilla call options (suitable 9 ways of creation) or 4 types of barrier call options (suitable 16 ways of creation) ensuring the maximum buying price for hedgers subjects, as we will see later.

The motivation of this paper is twofold. First, in order to propose various hedging variants through Short Call Ladder strategy using barrier options in analytical expression. Second, these models are applied to the energy markets to build risk management strategies in this market. There is important to determine optimal options strategies to suit for risk management in the market. The purpose of this strategy is to manage energy price changes, which may have a significant influence on buying price with the possibility of reducing costs. Our theoretical results of the hedged portfolios against a price increase can be applied to any underlying assets as shares, commodities, currencies or interest rates. Analyses the hedging effectiveness of different proposed variants are compared with each other and with the naked (unsecured) position. Based on our results, the recommendations for firms, which variant is the best in different underlying price development, are given.

The remainder of the paper is structured as follows. The next section presents the research methodology. In Section 3, the analysis of all hedging variants using Short Call Ladder strategy by barrier options is given and the models used in the paper discussed. Section 4 proposes the application in energy market, where the results and the findings are discussed. The closing section offers a summary and conclusion.

## 2. RESEARCH METHODOLOGY

The nature of this paper's approach is based on option strategies and their using on risk management. Options and option strategies represent the significant part of financial engineering, which are also used on the creation of new innovative financial products as it is proved by Hull (2012). The methodology of the paper assumes an analytical expression of the vanilla and barrier options' profit function.

Understanding of the Short Call Ladder strategy creation is needed for its using on hedging. Short Call Ladder strategy is created by selling  $n$  of call options with a strike price  $X_1$ , premium  $c_{1S}$  per option and at the same time by buying  $n$  of call options with the higher strike price  $X_2$ , premium  $c_{2B}$  per option and by buying  $n$  of call options with the highest strike price  $X_3$ , premium  $c_{3B}$  per option, where European-style of options for the same underlying asset and with the same expiration time is used.

Short Call Ladder strategy created by vanilla options and its using on hedging is dealt by Amaitiek et al. (2010). Also, Rusnáková and Šoltés (2012), Rusnáková (2015), Šoltés and Rusnáková, (2012; 2013) deal with the hedging against a price increase or drop by means of different options strategies using vanilla and barrier options. Following the studies mentioned we analyse all possible ways of Short Call Ladder strategy creation using barrier options with the aim to hedge against a price increase.

### 2.1 Data and summary models

For our approach, all Short Call Ladder hedging variants against a price increase are investigated. However, only suitable hedging portfolios with using of barrier options are analyzed in this paper. Only combinations of up and knock-in call options with the highest strike price fulfill our requirements for hedging. It is possible to create 9 types of hedging portfolios as combinations of up and knock-in call options together with vanilla call and barrier call options and 16 types as combinations of only barrier options, i.e. up and knock-in call options together with barrier call options. Analyzed ways have to fulfill the conditions of zero-cost option strategies creation.

European vanilla and barrier options on the Energy Select Sector SPDR ETF are used with various strike prices and the barrier levels. The vanilla option prices are real data obtained from [www.finance.yahoo.com](http://www.finance.yahoo.com). European barrier option prices are not publicly accessible, due to this fact we have calculated these prices. Black and Scholes (1973) introduced basic, generally used, option pricing model and their work is considered as significant added value in financial engineering theory and practice. This model cannot be used for the pricing of the barrier options. Merton (1973) modified classic version of this model for European down and knock-out call option. Later Rubinstein and Reiner (1991) applied the formulas for eight types of the barrier options and Haug (2007) for all 16 types of European style of the barrier options. According to Haug model (2007) we compute theoretical price of standard European barrier call options  $c$ , where the notations are defined as the actual underlying spot price  $S_0$ , the barrier level  $B$ , the strike price  $X$ , compensation  $K$ , the risk-free interest rate  $r$  (derived from government bonds yields – U.S. Treasury rate, source: [www.bloomberg.com](http://www.bloomberg.com)), the implied volatility  $\sigma$  (we used historical volatility), dividend yield  $q$  and the time to maturity of the option  $t$ . All derived relations of barrier call options can be found in Iacus (2011). Our calculations of selected barrier options

were processed in the statistical program R. Also, the barrier options can be priced by lattice techniques such as binomial (Cox et al., 1979) and trinomial trees (Ritchen, 1995) or Monte Carlo simulation (Boyle, 1977). New pricing method of exotic options was discussed in the paper (Nishiba, 2013).

### 3. ANALYSIS OF RISK MANAGEMENT WAYS USING BARRIER OPTIONS

In this part, we have analysed theoretical models' effectiveness of different hedging variants against price increase. We hedge a spot position of the underlying asset using Short Call Ladder strategy created by barrier options. All possible created scenarios are considered with the finding of the best way for hedging to the maximum buying price in the future.

Let us suppose that at time  $T$  in the future we will buy  $n$  pieces of the underlying asset, but we are afraid of its price increase. The profit function from the buying of the unsecured position at time  $T$  is:

$$P(S_T) = -n \cdot S_T, \quad (1)$$

where  $S_T$  is the underlying spot price at time  $T$ . Generally, the higher the underlying spot price is, the higher costs, which we have to pay for buying of the underlying asset, are. Our analysis shows that only using up and knock-in (UI) call options with the highest strike price are suitable for hedging against the price increase. Other types of call barrier options (UO, DI, DO) with the highest strike price secure the buying price only partially, therefore we have excluded these call options from our analysis. There are 25 types of hedging variants in total, which can be created using UI call options with the highest strike price together with the vanilla call or others barrier call options. In the next part we introduce only selected hedging variants fulfilling the condition of the zero-cost option strategies, i.e. the selling option premium should be higher or equal than sum of buying option premiums according to the following relation:

$$c_{1S} \geq c_{2B} + c_{3B} \quad (2)$$

The lower barrier levels  $D$  for call options can be set at  $D \leq X$  or  $D > X$ , but it is valid  $D < S_0$ . However, the upper barrier levels should be set as  $U > X$  and  $U > S_0$ .

At first, let us construct Short Call Ladder Strategy by selling  $n$  of call options with a strike price  $X_1$ , the premium  $c_{1S}$  per option and at the same time by buying  $n$  of call options with the higher strike price  $X_2$ , premium  $c_{2B}$  per option and by buying  $n$  of up and knock-in call options with the highest strike price  $X_3$ , premium  $c_{3BUI}$  per option and the upper risk bounds  $U$ , where the choice of the call strike prices should be given as  $X_1 < X_2 < X_3$ . Further, we assume that the upper barrier  $U$  is set above both the strike price  $X_3$  and the actual spot price at time of issue  $S_0$ , i.e.  $U > X_3$  and  $U > S_0$  and the expiration period is the same for all options. The profit functions from selling  $n$  of call options

$$P_1(S_T) = \begin{cases} n \cdot c_{1S} & \text{if } S_T < X_1, \\ -n \cdot (S_T - X_1 - c_{1S}) & \text{if } S_T \geq X_1, \end{cases} \quad (3)$$

from buying  $n$  of call options

$$P_2(S_T) = \begin{cases} -n \cdot c_{1S} & \text{if } S_T < X_2, \\ n \cdot (S_T - X_2 - c_{1S}) & \text{if } S_T \geq X_2, \end{cases} \quad (4)$$

and from buying  $n$  of up and knock-in call options

$$P_3(S_T) = \begin{cases} -n \cdot c_{3BUI} & \text{if } S_T < X_3, \\ n \cdot (S_T - X_3 - c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \geq U \wedge S_T \geq X_3, \\ -n \cdot c_{3BUI} & \text{if } \max_{0 \leq t \leq T} (S_t) < U \wedge S_T \geq X_3, \end{cases} \quad (5)$$

are shown in an analytical expression for easier way of understanding the created hedging strategies. According to previous conditions, the secured profit function 1 (6) is created as combination of the naked position (1) together with individual options (3), (4) and (5).

$$SP_1(S_T) = \begin{cases} -n \cdot (S_T - c_{1S} + c_{2B} + c_{3BUI}) & \text{if } S_T < X_1, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2B} + c_{3BUI}) & \text{if } X_1 \leq S_T < X_2, \\ -n \cdot (S_T - X_1 + X_2 - c_{1S} + c_{2B} + c_{3BUI}) & \text{if } X_2 \leq S_T < X_3, \\ -n \cdot (S_T - X_1 + X_2 - c_{1S} + c_{2B} + c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) < U \wedge S_T \geq X_3, \\ -n \cdot (X_3 + X_2 - X_1 - c_{1S} + c_{2B} + c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \geq U \wedge S_T \geq X_3. \end{cases} \quad (6)$$

It is valid, the higher the strike price is, the lower call option premiums are and vice versa. Taleb (1997) have proved that the vanilla option premiums are higher than the barrier option premiums, therefore the ability to exercise the barrier options is dependent on breaking the pre-set barrier. The hedging variants with using of barrier options introduce the most interesting way of option strategy creation, which secure the lowest costs from the buying of the underlying asset. Due to this fact, zero-cost option strategy is fulfilled in all created hedging variants. By comparison of the secured profit function (6) with the unsecured profit function (1) at the time  $T$ , we can conclude the following statements:

- We assume for our hedging purpose, that the upper barrier  $U$  will be reached during its time to maturity and  $S_T \geq X_3$  at the expiration date. Fulfillment of these two conditions, the maximum buying price is constant to level  $X_3 + X_2 - X_1 - c_{1S} + c_{2B} + c_{3BUI}$ . By comparing with the unsecured function, the costs will be lower with hedging strategy if  $S_T \geq X_3 + X_2 - X_1 - c_{1S} + c_{2B} + c_{3BUI}$ .
- If the barrier is not reached during time to maturity, but the underlying price is  $S_T \geq X_3$  or  $X_2 \leq S_T < X_3$ , the costs of buying an underlying asset are equal to  $S_T - X_1 + X_2 - c_{1S} + c_{2B} + c_{3BUI}$ , but these costs are higher in comparison with the unsecured position.
- If the underlying price is  $X_1 \leq S_T < X_2$ , then the cost of the hedging strategy will be  $2S_T - X_1 - c_{1S} + c_{2B} + c_{3BUI}$ , where lower costs of hedged position compared with the naked position are possible only for  $S_T < X_1 + c_{1S} - c_{2B} - c_{3BUI}$ .
- In the case of the underlying price  $S_T < X_1$ , the profit function will be  $S_T - c_{1S} + c_{2B} + c_{3BUI}$ , which means, that our costs for the last scenario would be lower than the unsecured position.

The more interesting hedging strategy is created by combination of selling  $n$  of call options with a strike price  $X_1$ , the premium  $c_{1S}$  per option and at the same time by buying one type  $n$  of following barrier call options

- down and knock-in call options with a strike price  $X_2$ , the premium  $c_{2BDI}$  per option, the barrier level  $D$ , where a lower barrier  $D < X_2$  and  $D < S_0$ ,

$$P_2(S_T) = \begin{cases} -n \cdot c_{2BDI} & \text{if } S_T < X_2, \\ n \cdot (S_T - X_2 - c_{2BDI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \leq D \wedge S_T \geq X_2, \\ -n \cdot c_{2BDI} & \text{if } \max_{0 \leq t \leq T} (S_t) > D \wedge S_T \geq X_2, \end{cases} \quad (7)$$

- down and knock-out call options with a strike price  $X_2$ , the premium  $c_{2BDO}$  per option, the barrier level  $D$ , where a lower barrier  $D < X_2$  and  $D < S_0$ ,

$$P_2(S_T) = \begin{cases} -n \cdot c_{2BDO} & \text{if } S_T < X_2, \\ n \cdot (S_T - X_2 - c_{2BDO}) & \text{if } \max_{0 \leq t \leq T} (S_t) > D \wedge S_T \geq X_2, \\ -n \cdot c_{2BDO} & \text{if } \max_{0 \leq t \leq T} (S_t) \leq D \wedge S_T \geq X_2, \end{cases} \quad (8)$$

- up and knock-in call options with a strike price  $X_2$ , the premium  $c_{2BUI}$  per option, the barrier level  $U$ , where an upper barrier can be  $U > X_2$  and  $U > S_0$ ,

$$P_2(S_T) = \begin{cases} -n \cdot c_{2BUI} & \text{if } S_T < X_2, \\ n \cdot (S_T - X_2 - c_{2BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \geq U \wedge S_T \geq X_2, \\ -n \cdot c_{2BUI} & \text{if } \max_{0 \leq t \leq T} (S_t) < U \wedge S_T \geq X_2, \end{cases} \quad (9)$$

and by buying  $n$  of up and knock-in call options with the highest strike price  $X_3$ , premium  $c_{3BUI}$  per option, the barrier level  $U$  and with the expiration period same as the other options.

General description of the secured profit strategy 2 (10) created as a combination of the unsecured position (1) together with the profit functions of the option positions (3), (7)/(8) and (5) and for the secured profit strategy 3 (11) created by (1), (3), (9) and (5) can be written as

$$SP_2(S_T) = \begin{cases} -n \cdot (S_T - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } S_T < X_1, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } X_1 \leq S_T < X_2, \\ -n \cdot (S_T - X_1 + X_2 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_1 \text{ is fulfilled } \wedge X_2 \leq S_T < X_3, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_2 \text{ is fulfilled } \wedge X_2 \leq S_T < X_3, \\ -n \cdot (S_T - X_1 + X_2 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_1 \text{ and } C_3 \text{ is fulfilled } \wedge S_T \geq X_3, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_2 \text{ and } C_3 \text{ is fulfilled } \wedge S_T \geq X_3, \\ -n \cdot (S_T - X_1 + X_3 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_2 \text{ and } C_4 \text{ is fulfilled } \wedge S_T \geq X_3, \\ -n \cdot (X_3 + X_2 - X_1 - c_{1S} + c_{2Bbarrier} + c_{3BUI}) & \text{if } C_1 \text{ and } C_4 \text{ is fulfilled } \wedge S_T \geq X_3 \end{cases} \quad (10)$$

$$SP_3(S_T) = \begin{cases} -n \cdot (S_T - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } S_T < X_1, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } X_1 \leq S_T < X_2, \\ -n \cdot (S_T - X_1 + X_2 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } C_1 \text{ is fulfilled } \wedge X_2 \leq S_T < X_3, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } C_2 \text{ is fulfilled } \wedge X_2 \leq S_T < X_3, \\ -n \cdot (2S_T - X_1 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } C_3 \text{ is fulfilled } \wedge S_T \geq X_3, \\ -n \cdot (X_3 + X_2 - X_1 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } C_4 \text{ is fulfilled } \wedge S_T \geq X_3. \end{cases} \quad (11)$$

The summary of barrier conditions for particular call barrier options is presented in Table 1. We get the profit function of the selected secured ways for Short Call Ladder strategy creation by substituting corresponding barrier conditions in general profit function.

**Table 1.** Buying of call barrier options

Type of put barrier option	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Conditions for barriers
down knock-in (DI)	$\min_{0 \leq t \leq T} (S_t) \leq D$	$\min_{0 \leq t \leq T} (S_t) > D$	$\max_{0 \leq t \leq T} (S_t) < U$	$\max_{0 \leq t \leq T} (S_t) \geq U$	$D \leq X_1$ or $D > X_1$
down knock-out (DO)	$\min_{0 \leq t \leq T} (S_t) > D$	$\min_{0 \leq t \leq T} (S_t) \leq D$	$\max_{0 \leq t \leq T} (S_t) < U$	$\max_{0 \leq t \leq T} (S_t) \geq U$	$D < S_0$
up knock-in (UI)	$\max_{0 \leq t \leq T} (S_t) \geq U_1$	$\max_{0 \leq t \leq T} (S_t) < U_1$	$\max_{0 \leq t \leq T} (S_t) < U_2$	$\max_{0 \leq t \leq T} (S_t) \geq U_2$	$U_1 = U_2$ $U_1 > X_2$ or $U_1 > X_3$ $U_1 > S_0$

Our last way is to show the creation of hedging variant using only barrier options. The best possibility is by selling  $n$  of up and knock-in call options with a strike price  $X_1$ , the premium  $c_{1BUI}$  per option, the barrier level  $U$ , which can be set as  $U > X_1$  and at the same time by buying  $n$  of down and knock-in call options with the higher strike price  $X_2$ , premium  $c_{2SDI}$  per option, the barrier level  $D$  and at the same time by buying  $n$  of up and knock-in call options with the highest strike price  $X_3$ , premium  $c_{3BUI}$  per option and the barrier level  $U$ . For easier showing, we assume the same upper barrier  $U$  for call options, i.e.  $U_1 = U_2$ .

The profit function of selling  $n$  of up and knock-in call options is

$$P_1(S_T) = \begin{cases} n \cdot c_{3BUI} & \text{if } S_T < X_1, \\ -n \cdot (S_T - X_3 - c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \geq U \wedge S_T \geq X_1, \\ n \cdot c_{3BUI} & \text{if } \max_{0 \leq t \leq T} (S_t) < U \wedge S_T \geq X_1, \end{cases} \quad (12)$$

Then the secured strategy 4 (13) formed by individual functions (1), (12), (7) and (5) can be written as:

$$SP_4(S_T) = \begin{cases} -n \cdot (S_T - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } S_T < X_1, \\ -n \cdot (S_T - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) < U_1 \wedge X_1 \leq S_T < X_2, \\ -n \cdot (2S_T - X_1 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \max_{0 \leq t \leq T} (S_t) \geq U_1 \wedge X_1 \leq S_T < X_2, \\ -n \cdot (S_T - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) > D \wedge \max_{0 \leq t \leq T} (S_t) < U_1 \wedge X_2 \leq S_T < X_3, \\ -n \cdot (X_2 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) \leq D \wedge \max_{0 \leq t \leq T} (S_t) < U_1 \wedge X_2 \leq S_T < X_3, \\ -n \cdot (2S_T - X_1 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) > D \wedge \max_{0 \leq t \leq T} (S_t) \geq U_1 \wedge X_2 \leq S_T < X_3, \\ -n \cdot (S_T - X_1 + X_2 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) \leq D \wedge \max_{0 \leq t \leq T} (S_t) \geq U_1 \wedge X_2 \leq S_T < X_3, \\ -n \cdot (S_T - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) > D \wedge \max_{0 \leq t \leq T} (S_t) < U_2 \wedge S_T \geq X_3, \\ -n \cdot (S_T - X_1 + X_3 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) > D \wedge \max_{0 \leq t \leq T} (S_t) \geq U_2 \wedge S_T \geq X_3, \\ -n \cdot (X_2 - c_{1SUI} + c_{2BDI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) \leq D \wedge \max_{0 \leq t \leq T} (S_t) < U_2 \wedge S_T \geq X_3, \\ -n \cdot (X_3 + X_2 - X_1 - c_{1S} + c_{2BUI} + c_{3BUI}) & \text{if } \min_{0 \leq t \leq T} (S_t) \leq D \wedge \max_{0 \leq t \leq T} (S_t) \geq U_2 \wedge S_T \geq X_3 \end{cases} \quad (13)$$

The maximum buying price is secured at the level  $X_3 + X_2 - X_1 - c_{1S} + c_{2B} + c_{3BUI}$  only if the lower barrier  $D$  and upper barrier  $U$  is reached during time to maturity. However, the choice of the vanilla call or barrier call options for investors is connected with their expectations on underlying asset's price development, i.e. if there is expected rapid/slow drop or rapid/slow increase.

This section has presented some theoretical hedging variants using Short Call Ladder strategy created by barrier options for the risk management framework. Based on these theoretical statements, specific practical examples are discussed in the next section.

## 4. APPLICATION IN ENERGY SECTOR

In this section, we apply our proposed hedging variants in energy sector using exchange traded funds (ETFs). ETFs are one of the interesting way how to invest in commodity market, mainly to energy. For implementing we have chosen Energy Select Sector SPDR ETF (XLE), because it is the largest fund in this category.

Let us assume, that we are planning to buy stocks of XLE in future (January 2018) in amount of 100 pieces, but we are afraid of its price increase. Therefore, we are planning to hedge through Short Call Ladder Strategy formed by vanilla and barrier options to limit the maximum buying price in future. The hedging variants has to fulfil the condition of the zero-cost option strategy at the time of issue according to relation (2), otherwise there are excluded from our observation. All ways of hedging variants are considered and compared. However, there are presented only selected possibilities, which we consider an interesting for hedgers.

### 4.1 Data description

On 18<sup>th</sup> March 2016 the Energy Select Sector SPDR ETF were traded at 63.49 USD per share. All options considered here are European options on XLE from 18<sup>th</sup> March 2016 and with the maturity date 19<sup>th</sup> January 2018. The dataset of vanilla call option premiums are obtained from finance.yahoo.com. Key hedging information is introduced in Table 2 and selected European call option premiums in Table 3.

**Table 2.** Key hedging information

<i>Underlying asset</i>	Energy Select Sector SPDR ETF (XLE)
<i>Issue date</i>	18 <sup>th</sup> March 2016
<i>Issue price</i>	\$ 63.49
<i>Maturity date</i>	19 <sup>th</sup> January 2018
<i>Multiplier</i>	1:1
<i>Dividend yield</i>	3.97%

**Table 3.** European call option premiums on 18<sup>th</sup> March 2016

Implied volatility (%)	Bid (USD)	Ask (USD)	Strike (USD)
26.71	18.4	20.25	45
24.65	14.85	16.1	50
22.83	11.4	12.3	55
22.00	8.35	9.2	60
21.28	4.75	5.5	68
20.73	4.05	4.65	70
21.70	2.64	3.6	75

Source: finance.yahoo.com

European barrier call options are calculated in statistical program R, where the input parameters used are the spot price of the underlying asset (63.49 USD), time to maturity (673 days, i.e. hedging period from 18<sup>th</sup> March to 19<sup>th</sup> January 2018), interest rate 0.8365% (gained from treasury.gov), dividend yield 3.97% (gained from google.com/finance) and historical volatility 0.2542%. Due to simplifications, transaction costs and trading constraints are not taken into consideration.

The dataset for our analysis consists of 38 vanilla call options, 50 UI/UO call options and 50 DI/DO call options. Currency of an underlying asset and the option premiums is USD. There are considered the strike prices of real vanilla call options (gained from finance.yahoo.com) in the range of 45 – 78. The barriers were selected by authors, for lower barriers of DI/DO options in the range of 20 – 60 and for upper barriers of UI/UO options in the range of 70 – 110, all in the multiplies of 10. On the basis of dataset, there are analyzed the hedging variants with the strike prices:

$$X_1 = 45, X_2 = 60 \text{ and } X_3 = 70,$$

$$X_1 = 50, X_2 = 68 \text{ and } X_3 = 70,$$

$$X_1 = 55, X_2 = 70 \text{ and } X_3 = 75,$$

and considered the lower barrier  $D$  (20, 30, 40, 50 and 60) and upper barrier  $U$  (80, 90, 100, and 110). There is totally 4788 hedging variants proposed. Not all results obtained from our testing are presented in the paper, because we choose only selected variants fulfilling above stated requirements.

## 5. RESULTS AND DISCUSSION

The hedging against a price increase have secured a maximum acceptable underlying asset price. Therefore, the zero-cost option conditions (2) are preferred for our proposed hedging variants. The actual spot price is 63.49 USD and we expect its price increase in future  $T$ . For easier way, we have considered the same upper barrier  $U$  for UI call options in the level of 90 USD.

1. According to the first hedging variant, we will sell 100 call options with a strike price  $X_1 = 50$ , premium  $c_S = 14.85$  per option and at the same time, we will buy 100 call options with the strike price  $X_3 = 68$ , premium  $c_B = 5.50$  per option and at the same time we will buy 100 up and knock-in call options with the strike price  $X_3 = 70$ , the barrier level  $U = 90$ , premium  $c_{BUI} = 4.53$ . The hedged profit function from the purchase of 100 shares is given as

$$SP_1(S_T) = \begin{cases} -100 \cdot S_T + 525.77 & \text{if } S_T < 50, \\ -200 \cdot S_T + 5525.77 & \text{if } 50 \leq S_T < 68, \\ -100 \cdot S_T - 1274.23 & \text{if } 68 \leq S_T < 70, \\ -100 \cdot S_T - 1274.23 & \text{if } \max_{0 \leq t \leq T} (S_t) < 90 \wedge S_T \geq 70, \\ -8274.23 & \text{if } \max_{0 \leq t \leq T} (S_t) \geq 90 \wedge S_T \geq 70. \end{cases} \quad (14)$$

This way of construction is the simplest of all. We have compared the functions (14) with the unsecured position (1) at various XLE price development during time to maturity, where the following conclusions are formulated:

- If the spot price of shares during time to maturity grows above upper barrier  $U = 90$  and is higher than 70 USD at the future time  $T$ , then this hedging variant secures the maximum costs in height of 82.74 USD per share.
- If the upper barrier level  $U = 90$  is reached during time to maturity and the spot price at time  $T$  is lower than 55.26 USD or higher than 82.74 USD, then our hedged variant is still better than the unsecured position. Otherwise the unsecured position is better.

2. From the point of view of the creation, the hedging variants with using of two barrier options are interesting for hedgers. We will sell 100 call options with a strike price  $X_1 = 50$ , premium  $c_S = 14.85$  per option and at the same time, we will buy 100

2A. DI call options

2B. DO call options

### 3. UI call options

with a strike price  $X_2 = 68$ , the barrier level  $D = 40$  (for DI and DO) or  $U = 90$  (for UI), premium  $c_{BDI} = 0.01$  for DI ( $c_{BDO} = 5.20$  for DO,  $c_{BUI} = 4.44$  for UI) per option and at the same time, we will buy 100 up and knock-in call options with the strike price  $X_3 = 70$ , the barrier level  $U = 90$ , premium  $c_{BUI} = 4.09$ . The hedged profit functions from the buying of 100 shares for first two alternatives are shown in the Table 4. The third hedged variant is expressed by equation (15).

**Table 4.** Comparison of the profit functions for hedging variants 2A , 2B

Scenarios of the spot price during time to maturity $t$ and at the maturity $T$	Hedging variant 2A	Hedging variant 2B
$S_T < 50$	$1,074.69 - 100 \cdot S_T$	$555.45 - 100 \cdot S_T$
$50 \leq S_T < 68$	$6,074.69 - 200 \cdot S_T$	$5,555.45 - 200 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) < 40 \wedge 68 \leq S_T < 70$	$-725.31 - 100 \cdot S_T$	$5,555.45 - 200 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) \geq 40 \wedge 68 \leq S_T < 70$	$6,074.69 - 200 \cdot S_T$	$-1,244.55 - 100 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) < 40 \wedge \max_{0 \leq t \leq T} (S_t) < 90 \wedge S_T \geq 70$	$-725.31 - 100 \cdot S_T$	$5,555.45 - 200 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) \geq 40 \wedge \max_{0 \leq t \leq T} (S_t) < 90 \wedge S_T \geq 70$	$6,074.69 - 200 \cdot S_T$	$-1,244.55 - 100 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) < 40 \wedge \max_{0 \leq t \leq T} (S_t) \geq 90 \wedge S_T \geq 70$	$-7,725.31$	$-1,444.55 - 100 \cdot S_T$
$\min_{0 \leq t \leq T} (S_t) \geq 40 \wedge \max_{0 \leq t \leq T} (S_t) \geq 90 \wedge S_T \geq 70$	$-925.31 - 100 \cdot S_T$	$-8,244.55$

$$SP_3(S_T) = \begin{cases} 631.73 - 100 \cdot S_T & \text{if } S_T < 50, \\ 5,631.73 - 200 \cdot S_T & \text{if } 50 \leq S_T < 68, \\ -1,168.27 - 100 \cdot S_T & \text{if } \max_{0 \leq t \leq T} (S_t) < 90 \wedge 68 \leq S_T < 70, \\ 5,631.73 - 200 \cdot S_T & \text{if } \max_{0 \leq t \leq T} (S_t) \geq 90 \wedge 68 \leq S_T < 70, \\ 5,631.73 - 200 \cdot S_T & \text{if } \max_{0 \leq t \leq T} (S_t) < 90 \wedge S_T \geq 70, \\ -8,168.27 & \text{if } \max_{0 \leq t \leq T} (S_t) \geq 90 \wedge S_T \geq 70. \end{cases} \quad (15)$$

Comparison of the above proposed three hedging variants (2A, 2B and 3) gives us the following results:

- The hedging variant 2A, created by the basic positions (call options, DI and UI call options), ensures the lowest maximum costs at expected intervals of the spot price at the time  $T$  in the amount of 77.25 USD per share. In this case the higher volatility of the shares is expected.
- If the lower barrier  $D$  and the upper barrier  $U$  is reached during time to maturity  $T$ , the hedging variant 2A is better than unsecured position and others variant for the spot price at time  $T$  lower than 60.75 USD and higher than 77.25 USD. Otherwise, the unsecured position is the best.
- If the lower barrier  $D$  is not reached and the upper barrier  $U$  is reached during time to maturity  $T$ , the hedging variant 3 ensures the lowest maximum costs in amount of 81.68 USD per share. However, if the spot price is lower than 60.75 USD, the hedging variant 2A is better, otherwise, the unsecured position from interval  $\langle 60.75; 81.68 \rangle$ .
- If the upper barrier  $U$  is not reached during time to maturity  $T$ , the unsecured position is still better from the level of 60.75 USD as others analysed hedging variants.

4. In the last case let us create Short Call Ladder strategy by selling of 100 UI call options with a strike price  $X_1 = 50$ , the barrier level  $U = 90$ , premium  $c_{SUI} = 7.95$  per option and at the same time by selling 100 DI call options with the strike price  $X_2 = 68$ , the barrier level  $D = 40$  premium  $c_{BDI} = 0.01$  per option and by buying 100 up and knock-in call options with the strike price  $X_3 = 70$ , the barrier level  $U = 90$ , premium  $c_{BUI} = 4.09$ . The hedged profit function from the buying of 100 shares is given as

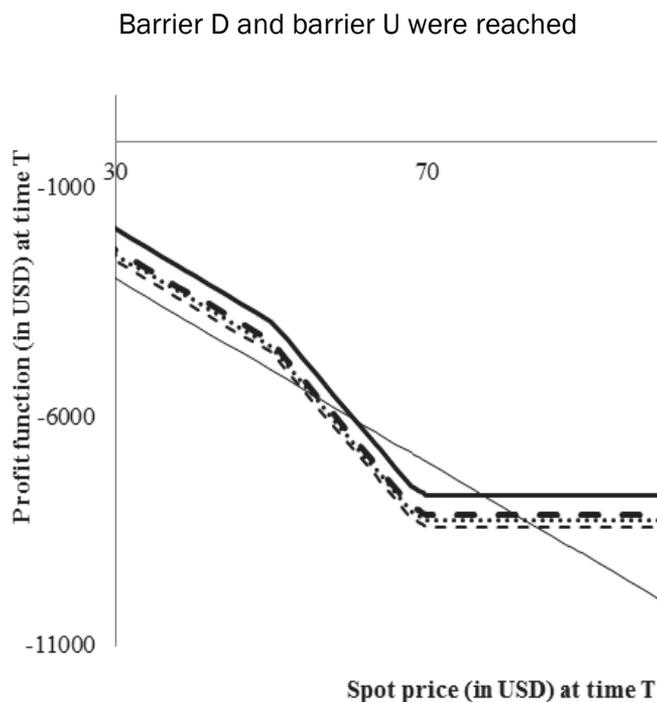
$$SP_4(S_T) = \begin{cases} -100 \cdot S_T + 384.80 & \text{if } S_T < 50, \\ -100 \cdot S_T + 384.80 & \text{if } \max_{0 \leq t \leq T}(S_t) < 90 \wedge 50 \leq S_T < 68, \\ -200 \cdot S_T + 5384.80 & \text{if } \max_{0 \leq t \leq T}(S_t) \geq 90 \wedge 50 \leq S_T < 68, \\ -100 \cdot S_T + 384.80 & \text{if } \min_{0 \leq t \leq T}(S_t) > 40 \wedge \max_{0 \leq t \leq T}(S_t) < 90 \wedge 68 \leq S_T < 70, \\ -6415.20 & \text{if } \min_{0 \leq t \leq T}(S_t) \leq 40 \wedge \max_{0 \leq t \leq T}(S_t) < 90 \wedge 68 \leq S_T < 70, \\ -200 \cdot S_T + 5384.80 & \text{if } \min_{0 \leq t \leq T}(S_t) > 40 \wedge \max_{0 \leq t \leq T}(S_t) \geq 90 \wedge 68 \leq S_T < 70, \\ -100 \cdot S_T - 1415.20 & \text{if } \min_{0 \leq t \leq T}(S_t) \leq 40 \wedge \max_{0 \leq t \leq T}(S_t) \geq 90 \wedge 68 \leq S_T < 70, \\ -100 \cdot S_T + 384.80 & \text{if } \min_{0 \leq t \leq T}(S_t) > 40 \wedge \max_{0 \leq t \leq T}(S_t) < 90 \wedge S_T \geq 70, \\ -100 \cdot S_T - 1615.20 & \text{if } \min_{0 \leq t \leq T}(S_t) > 40 \wedge \max_{0 \leq t \leq T}(S_t) \geq 90 \wedge S_T \geq 70, \\ -6415.20 & \text{if } \min_{0 \leq t \leq T}(S_t) \leq 40 \wedge \max_{0 \leq t \leq T}(S_t) < 90 \wedge S_T \geq 70, \\ -8415.20 & \text{if } \min_{0 \leq t \leq T}(S_t) \leq 40 \wedge \max_{0 \leq t \leq T}(S_t) \geq 90 \wedge S_T \geq 70. \end{cases} \quad (16)$$

This strategy, created only using barrier options, is the profitable than the others designed for the case of reaching the lower barrier and not reaching the upper barrier with the amount of 64.15 USD per share. In the case of unfavorable shares price development (reaching the lower  $D = 40$  and upper barrier  $U = 90$ ) the maximum buying price of 84.15 USD per share is secured, otherwise, the unsecured position for interval  $\langle 53.85; 84.15 \rangle$ . If significant increase and simultaneously drop are not expected (not reaching the barriers), then the hedged variant is still better than the unsecured position.

The graphical comparison (Figure 1) of all proposed hedging variants is expressed for all possible scenarios of the underlying price development. Using the analytical expressions of the profit functions (14), (from Table 3), (15) and (16) and the graphical expressions of 1, 2A, 3 and 4 secured possibilities (Figure 1), we can conclude, that:

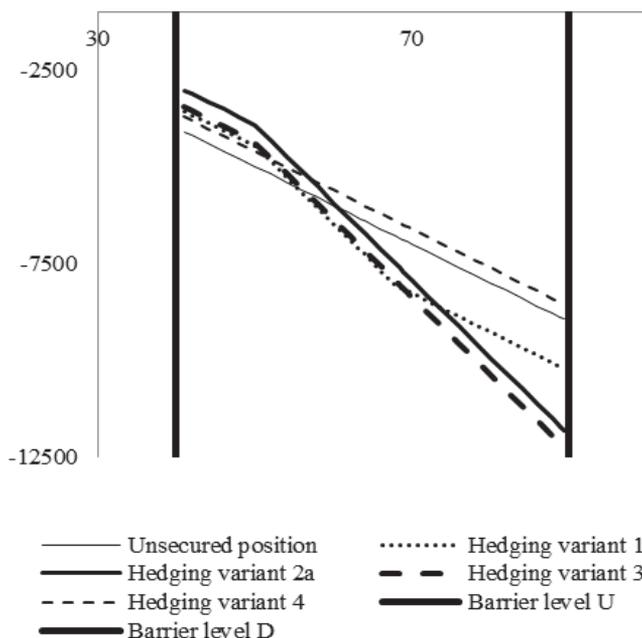
- If the spot price of shares during time to maturity falls under the lower barrier  $D = 40$ , at the same time does not grow above the upper barrier  $U = 90$  and at the maturity date is higher than 56.90, then the hedging variants 4 is still better than the others variant, otherwise variant 2A is better.
- If the upper  $U = 90$  and lower  $D = 40$  barrier levels was not reached during time to maturity, then there are the same conclusions as the previous one.
- If the upper  $U = 90$  and lower  $D = 40$  barrier levels are reached during time to maturity, then the hedging variant 2A is the best for intervals lower than 60.75 USD and higher than 77.25 USD, otherwise the unsecured position is better.
- But if the upper barrier  $U = 90$  is reached and the lower barrier  $D = 40$  is not reached during time to maturity and the spot price of shares is lower than 60.75 USD, then the variant 2A is better. Otherwise, if the spot price is higher than 81.68, the variant 3 is better, but the unsecured position is the best for interval  $\langle 60.75; 81.68 \rangle$ .

**Figure 1.** Comparison of the profit functions at time  $T$  of the hedging variants 1, 2A, 3 and 4





### Barrier D and barrier U



The results of our study reveal that the choice of the hedging variants using barrier options depends on hedger's expectations of underlying asset's price development. Fulfilment of the stated zero-cost option condition above (2), there is possible to decide between knock-in options and knock-out options. Knock-in options are chosen in significantly price increase/drop, when the option is activated after reaching the barrier level. Otherwise, knock-out options are chosen, when the option is deactivated after reaching the barrier, in moderately price increase/drop. All designed hedging variant have provided an interesting opportunity to secured maximum buying price of shares. Results of our analysis show that designed hedging variants for all scenarios of future underlying asset's price development are not suitable for better securing against the unsecured position.

Due to expectation of the price increase we focus only on reaching the upper barrier levels. In case of reaching both of barriers during time to maturity  $T$ , hedging variant 2A is the best solution for securing the underlying's price. On the other side, if the lower barrier is not reached, then the hedging variant 4 is the best. However, all investors should note that if the price at the future time  $T$  does not meet their expectations, they could gain a loss in comparison to the unsecured position.

## CONCLUSION

During the period of the high volatility, firms face to many challenges and risks. With the increasing of the risks, new opportunities are showed in the form of different tools suitable for the risk management. This paper investigated the optimal hedging strategy for a firm through Short Call Ladder strategy using barrier options, where the role of the basis risk are considered. To the best of our knowledge, the only little research that has investigated the risk management using barrier option strategies.

At the beginning the paper have provided the overview of the empirical studies following with research methodology of the option pricing techniques. Analytical expression of options were used for hedged portfolio creation in the increasing markets, where only selected hedg-

ing variants were introduced in the paper. For our purpose there were appropriate only up and knock-in call options with the highest strike price when the hedger wants to secure against increase. There is possible to create more hedging alternatives, but only 25 ways of Short Call Ladder strategy using barrier options are suitable for fully hedging, i.e. 9 types with both barrier and vanilla options and 16 types with only barrier options. Based on the investigation of hedging strategies' advantages and disadvantages, there were found suitable variants securing the most likely unfavorable future price movement scenarios. The results of our study reveal that barrier options are better tool for securing the underlying asset's price.

The main contribution of this study is to jointly analyse and compare different Short Call Ladder strategy creation using barrier options with the application in energy sector. There is found that the buying price of the underlying asset is primarily affected by the amount of cash spent on the hedging. Hence, the primarily aim is to minimize the hedged portfolio costs, which is also meet by requirements of zero-cost option strategy. Due to this fact, hedgers do not have to pay any option premium in the beginning. Followed the mentioned assumptions, only best variants are introduced. Then, numerical examples are illustrated through analytical expression of Short Call Ladder strategy using barrier options with main aim of risk reduction. According to our analysis we recommend the hedging variant 2A or 4 as the best variant ensuring the lowest costs at expected intervals of the shares spot price at the maturity. Also, others designed variants were not excluded from our investigation, but were less interesting. The construction of option strategies are taken fully consideration of future uncertainty on underlying asset's price development. Therefore, it is important to select suitable combinations of the strike prices, the lower and the upper barriers for the best hedging profit function's achievement.

The paper demonstrated the significance of hedging for firms, which are planning to buy some underlying asset in the future. The methodology presented in this paper can be adapted to any market. On the other hand, hedgers have to realize the expectations of the energy price development as well as the willingness to undertake a risk, before they make a final decision about the choice of standard call/barrier call options.

Finally, this paper was focused on firms as hedgers, but this approach could be useful for others as well, such as financial institutions or academicians. Also, our approach can be helpful for various hedging option strategies creation in different financial markets.

## ACKNOWLEDGMENT

This paper is the partial result of the Ministry of Education of Slovak Republic grant project VEGA Nr. 1/0986/15 - Proposal of the dimensional models of the management effectiveness of ICT and information systems in health facilities in Slovakia and the economic.

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