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OPTIMALITY AND EQUILIBRIUM IN A SINGLE-PRODUCT ECONOMIC MODEL WITH COLLECTIVE GOOD (COMPUTER EXPERIMENTS)

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ABSTRACT

This paper considers some of the simplest models of the economy with the collective good. Used computer simulations illustrate the relationship between equilibrium and the optimum. We propose a heuristic method for finding the economic equilibrium with the collective goods, which is a variant of the tâtonnement process.

KEY WORDS: *computer simulations, collective good, equilibrium, optimum, tâtonnement process.*

JEL classification: C61, C62, C63, C72
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1. INTRODUCTION

Models of using collective goods initially emerged to describe simple situations where the same goods are consumed simultaneously by different individuals who have different preferences (joint purchase of the TV, smoking in a public place, gatecrasher in public transport, etc.) [2;7]. Generalization of these models to the case of models of economic equilibrium, and others has borne to the present time numerous publications. The situations with several kinds of collective goods have been considered, these goods can include the production function, can be measured by discrete values, can consider the role of the state, etc. etc. [8;3]. Much attention is paid to the analysis of game-theoretic features of models, Pareto optimal equilibrium and the process of equilibrium tâtonnement. [3]. In the context of experimental economics studies the behavior of people living in situations similar to the process of formation on private manner social benefits. [1]

Our work does not deal with complex mathematical structures and abstract schemes. It built and analyzed a simple mathematical economics equilibrium model, in which along with the private consumption of the product produced in the system, is created and consumed the public (collective) product. Important lever of regulation of economic relations is the tax policy of the state, and this effect on the equilibrium should be considered in the economic and mathematical models [4]. The paper discusses several ways of forming the rules of formation of taxes (see also [5]). The main goal - to show with some simple computer models, how an economic system can behave, seeking for equilibrium, and how it deviates from the Pareto-optimal state.

Process of simulation will represent the interaction of the market with a certain Center, with specification certain corrective parameters (taxes, government subsidies, and sometimes part of the collective good, etc.) and fixing total "burden" (defense, balance of exports and imports, etc.). Modeling of this process is based on two ideas. The first is that taxes on the participants can be given not only to fixed shares total planned budget, but under a special rule that (in some cases) provides a "social justice".[6] The second idea is that each participant performing their investment to the collective goods assumes that others will do as they did before. Center behaves similarly, predicting some general characteristics of the market process. This is explained in detail.

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2. BASIC MODEL RELATIONS.

Main relation of equilibrium:

$$y - \sum_k x_k - \sum_k F_k - Z \geq a\ , \qquad (1)$$

where $y = y\,(l)$ - production volume depending on the amount of labor l, k - numbers of partici-
pants
(groups) , and consuming the product involved in the production (k = 1,2, ... n).
Where:

x_k - consumption of private goods to k-th participant,
l_k - labor costs to k-th participant,
F_k - investments in the collective good to k-th participant,
Z - government investment in the public good,
a_k - factor of comparing the usefulness of collective good and usefulness labor consumption,
a - fixed burden on the economy.

The utility function (concave) k-th participant

$$u_k = v_k(x_k, l_k) + a_k \cdot \Phi(Z + \sum_k F_k) \qquad (2)$$

As can be seen from the formula (2), the usefulness of collective goods for participants is comparable with usefulness labor-consumption, but with different coefficients.

Equilibrium is defined as the set of behavior variables of all participants l_k^*, x_k^*, F_k^*, Z^*), the price p^*, wages q^* and tax requirements prescribed by Center D_k^*, satisfying the four conditions.

I. Natural Balance:

$$y^* - \sum_k x_k^* - \sum_k F_k^* - Z^* = a\ ,$$

II. Maximizing profit by producer

$$\Pi^* = p^* \cdot y^* - q^* \cdot \sum_k l_k^* = \max\ ,\ \ y=f(l)\ ,\ l<=\Sigma\, l_k.$$

III. Maximizing utility function by consumer

$$u_k(x_k^*, l_k^*, \sum_k F_k^*) \geq u_k(x_k^*, l_k^*, F_k + \sum_{i \neq k} F_i^*)\ ,\ \ i \neq k,$$

under the budget constraint

$$p \cdot x_k \leq q^* \cdot l_k - p^* \cdot F_k - D_k^*,$$

IV value balance (Walras law), П * - profit:

$$\sum_k D_k^* + \Pi^* - p^* \cdot a = 0\,.$$

Note that the equilibrium I-IV is a typical point of the corresponding Nash game, as his utility function depends on the choice of the other participants. Further, several variant of models will be discussed.

Model 1. Equilibrium, when the Center is not involved in the creation of the collective good *(Z = 0)* by determining only the tax requirements.

Model 2. In the constraint (*1*) and in the argument of the utility function *(2)* $F_j = 0$ for all $j \neq k$. Taxes to k-th user defined as a fixed percentage of the total budget, and the collective good is formed by the participants in the market and the Centre, so that each participant has a collective good, consisting of his own contribution and the same for all the Centre's contribution.

Model 3. Center forms a general tax on the basis of the behavior of market participants in previous time (by value balance) .Taxes on the participants in the first and third variant are formed in a manner that ensures compliance with the principle of justice in the form of a utilitarian social welfare function with constant coefficients of comparing interest when there are no social benefits [7, 8] . This method consisted in the fact that the Centre, specifying the proportion of taxation, continuously determines by some standard λ_k the amount of taxes, providing equality

$$\lambda_k \cdot \beta_k(D_k) = \lambda_i \cdot \beta_i(D_i) \text{, for all } j \neq k, \qquad (3)$$

where D_k, D_i - the amount of taxes, β_k - the marginal utility of household income, λ_k - coefficients of utility comparison of public welfare (*w*). This ensures that system enters in equilibrium, maximizing *w* (without collective goods).

$$w = \sum_k \lambda_k \cdot u_k(x_k, l_k, F) \qquad (3a)$$

with (1) - (5).

3. ANALYSIS OF THE MODELS

Model 1. Center is not involved in the creation of a collective good. The utility function of participants has the form

$$u_k = v_k(x_k, l_k) + a_k \cdot \Phi(\sum_k F_k)$$

Value balance and natural one (without Z) - similar to the former. Let us to analyze the situation at the equilibrium point obtained with the fulfillment of the condition (3) in the general case and compares it with the optimal solution. Optimization to k-th user its utility function, at the equilibrium point, leads to the equations:

$$\frac{dv_k}{dx_k} = a_k \cdot \frac{d\Phi}{dF_k} = \beta_k^* \cdot p^* \qquad (4)$$

where β^*_k - the marginal utility of income. For any equilibrium can choose coefficients λ_k so that $\beta^*_k \lambda_k = H = const$.

We now show that the solution (x_k^*, l_k^*, F_k^*) is optimal for *W* c coefficients λ_k, only when $a_k\lambda_k = a_i \lambda_i$. for all *i, k.*

Indeed, the optimum point must be satisfied:

$\lambda_k \cdot \frac{dv_k}{dx_k} = p^0$, – where p^0 - Lagrange multiplier to balance constraints (1) and at the equilibrium

point $\frac{dv_k}{dx_k} = p^* \cdot \beta_k^* = a_k \cdot \frac{d\Phi}{dF_k}$ for all *k*. Since all partial derivatives $\frac{d\Phi}{dF_k}$ in the optimum point are

equal, then the products $\lambda k \cdot ak$ must be constants. Also see that $p^0 = Hp^*$.

On the other hand, if in the equilibrium model, all these products $\lambda_k \cdot a_k$ equal, then (with $\beta^*_k \lambda_k = const$) multiplying all the coefficients λ_k for some constant, we obtain the equality of derivatives $\frac{dv_k}{dx_k}$, that the logarithmic utility functions implies equality of *x*-s themselves. Calculations on the model are given in Annex 2.

When modeling entrances of the economy in equilibrium (the process of "groping"), we have assumed that at each step Center trace for fulfillment of equality (3). This procedure in the absence of collective good results in an equilibrium that maximize social welfare *W* by these coefficients λ_k and at the same balance constraint. We are finding the optimal solution when determining the value of all the undivided public good *F* (see Table 1.2). For the problem

$$y - x - F = a, \quad W = max, \tag{7}$$

we write the Lagrange conditions when in balance (*1*), and in the utility functions of centralized investments *Z* is absent, and the letter *F* denotes undivided amount of private investment in the collective good (we assume that the functions *v(x, l)* and *Φ(F)* - logarithmic).

$$(A1 \cdot l1^{\alpha 1} + A2 \cdot l2^{\alpha 2} - a) \cdot p = \lambda 1 \cdot (1 + a1) + \lambda 2 \cdot (1 + a2),$$

$$\frac{-\lambda 1 \cdot b1}{T1 - l1} + p \cdot A1 \cdot \alpha 1 \cdot l1^{\alpha 1 - 1} = 0,$$

$$\frac{-\lambda 2 \cdot b2}{T2 - l2} + p \cdot A2 \cdot \alpha 2 \cdot l2^{\alpha 2 - 1} = 0,$$

$$\frac{\lambda 1 \cdot a1 + \lambda 2 \cdot a2}{F} \le p.$$

Letter *p* denotes a Lagrange multiplier (the "shadow price") produced and consumed product. Consumption of participants is as follows

$$x1 = \frac{\lambda 1}{p}, \quad x2 = \frac{\lambda 2}{p}.$$

The total value of the collective good is

$$\frac{\lambda 1 \cdot a1 + \lambda 2 \cdot a2}{p} = F$$

It is quite obvious that arbitrarily breaking between participants this value (in the absence of the market), we obtain the state of the economic system (y^0, x_k^0 , l_k^0, F_1^0, F_2^0), which maximizes the function *W* – in contrast to equilibrium. In other words, the market equilibrium (in the language of model 1) is less efficient in terms of the social welfare function if products $\lambda_k \cdot a_k$ is different.

We write down the expressions for the parameters of rational behavior of households in the market conditions, with the presence of the collective goods, derived from the conditions for the Lagrange

$$u_k = ln(x_k) + b_k \cdot ln(T_k - l_k) + a_k \cdot ln(z_1 + z_2),$$

where *p, q1, q2* - the price of the product and work evaluation; *D1, D2* - tax payments to the budget, *x1, z1, x1, z2* - private consumption and contribute to the collective good.

$$x1 = \frac{T1 \cdot q1 - D1 + p_t \cdot z2}{p \cdot (1 + a1 + b1)} \qquad x2 = \frac{T2 \cdot q2 - D2 + p \cdot z1}{p \cdot (1 + a2 + b2)}$$

$$l1 = T1 - \frac{b1 \cdot (T1 \cdot q1 - D1 + pz2)}{q1 \cdot (1 + a1 + b1)} \qquad l2 = T2 - \frac{b2 \cdot (T2 \cdot q2 - D2 + pz1)}{q2 \cdot (1 + a2 + b2)} \tag{8}$$

$$z1 = \max\left[\frac{a1 \cdot [T1 \cdot q1 - D1 + p \cdot (z2)]}{p \cdot (1 + a1 + b1)} - z2, 0\right] \qquad z2 = \max\left[\frac{a2 \cdot [T2 \cdot q2 - D2 + p \cdot (z1)]}{p \cdot (1 + a2 + b2)} - z1, 0\right]$$

$$\beta_1 = \frac{1 + a1 + b1}{T1 \cdot q1 + z2 \cdot p - D1} \qquad \beta_1 = \frac{1 + a2 + b2}{T2 \cdot q2 + z2 \cdot p - D2}$$

Further provides the results of computer calculations of equilibrium based on formulas and optimum. In Table 1, the parameters imposed condition $\lambda_k \cdot a_k = \lambda_2 \cdot a_2$. We see that all physical indicators coincide. Between price indices (see Table 2) remains constant ratio 0.904. If you violate the condition of product $\lambda_k \cdot a_k$, all values of indicators - different. We can assume that the deviation values of parameters these equations in some sense characterizes the difference between equilibrium and optimality.

Table 1: Model parameters

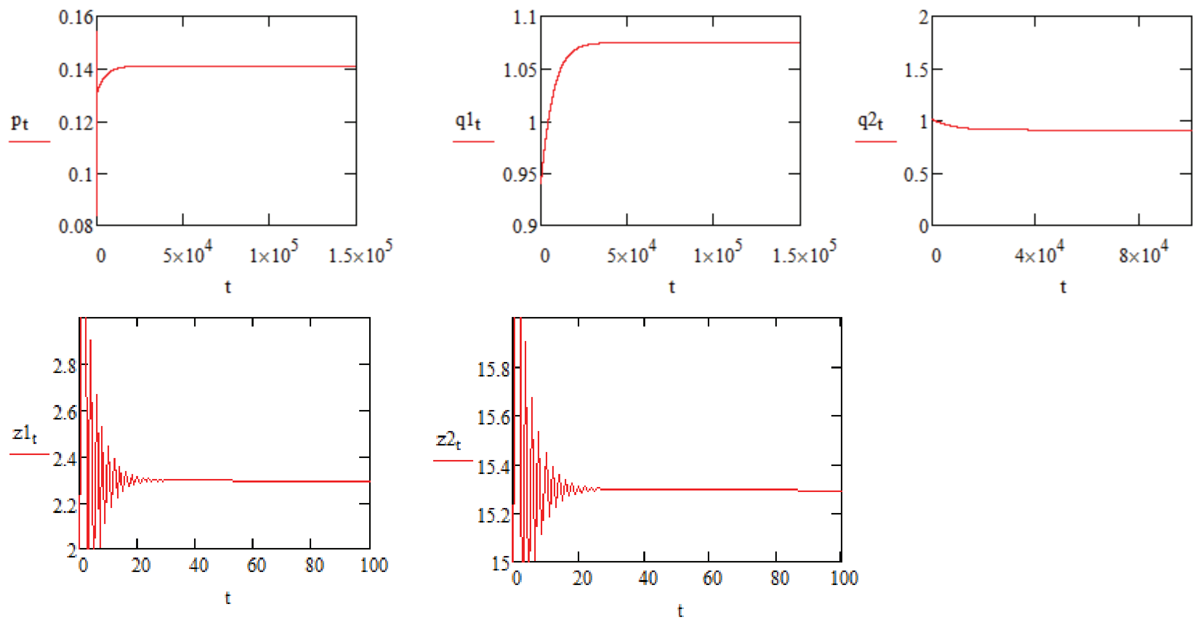
a	a_k	b_k	a_k	T_k	α_k	λ_k
75	0.258	0.156	11	16	0.9	9.5
75	0.31	0.23	10	14	0.88	7.9

Table 2: Found model indicators

	X1	X2	l1	l2	y1	y2	F1+F2	p	q1	q2
Equilibrium	65.84	54.7	14.64	12.55	123.2	89.42	16.98+0	0.144	1.092	0.942
Optimum	59.53	49.50	14.77	12.24	124.1	90.6	30.	0.16	1.207	1.04

As mentioned above, for the calculation of equilibrium an iterative process was used, so that at each time t values leading price variables p_t, $q1_t$, $q2_t$, $D_{k,t}$ and the value $x_{k,t}$, $y_{k,t}$, $l_{k,t}$, $F_{k,t}$ are determined by the "law of supply and demand." Maximization of consumer utility function runs on the assumption that others do not change their last choice. (In Annex 1, we specifically pointed way of finding solutions of the game when players making choices, are guided by previous decisions of the others). Similarly act and the Centre, setting the general tax level D, such as it was formed in the previous moment. Computer calculations given here are actually an example of heuristic modeling, as we used in the calculation formula (3), which as has been proven guarantee access to the equilibrium in the absence of collective goods. Iterative finding solutions of games (Nash point) by using information on the previous choice of the other participants are heuristic (cf. the results of calculations in annex 1)....

Figure 1: The trajectories of convergence to equilibrium performance model 1



Model 2. Taxes to k-th user defined as a fixed percentage of the total budget, and the collective good is formed by the participants in the market. Z value of centralized investments in collective good is formed by the rule, which actually performs while optimizing Model 1 (see above formula for *F*).

The behavior of each market participant is to maximize the function (2) (with logarithms) under conditions provided $px=ql-D-pF_k$\`\`. Manufacturer, as usual, maximizes profit. Centre appoints taxes D_k, focusing on the value of total tax revenues D previous moment of time: $D_k = \psi_k D$. Here the rule (3) does not work. For completeness of comparison "conditionally optimal" state is calculated by implementing λ-coefficients, what is inversely proportional to the marginal utility of income equilibrium.

Former algorithm entrance in equilibrium for two participants and logarithmic utility functions leads to a state of the economy, with the variables, which do not reveal any regular connection as "optimal".

The results of the calculations are presented in the tables below.

Table 3: Model 2 parameters

a	a_k	b_k	A_k	T_k	α_k	λ_k
35	0.5	0.156	15	16.1	0.9	21.577
35	0.581	0.28	18	14	0.88	17.504

The resultant state of the market economy, as noted earlier, is not Pareto optimal. As the table shows, the distribution of the tax burden between the two parties on shares ψ_k and $1-\psi$ leads to a condition significantly different from optimal.

Table 4: Founded Figures Model 2

	X1	X2	l1	l2	y1	y2	F1+F2	F	p	q1	q2
Equilibrium	122.19	99.12	14.26	11.65	163.96	156.16	6.2+2.7	54.87	0.177	1.83	2.08
Optimum	104.82	85.08	14.5	11.9	166.64	160.02	0	101.8	0.206	2.13	2.42

As can be seen from Table 4, the resulting "optimum" is characterized by increased labor costs, reduction in private consumption, but growing volumes of collective consumption.

Model 3. In the third version of the model, it is assumed that the Centre sets the value of its investments in the collective good, depending on the current prices by type of optimizing the overall collective good considered in accordance with these models.

$$Z = \frac{\lambda_1 \cdot a_1 + \lambda_2 \cdot a_2}{p_t}.$$

In addition, it determines the amount of general taxes, setting it equal to what it was at the previous moment. Further, it divides this value by the formula (3). It is clear that the multiplication of λ- coefficients to a positive value will not change the rules for the distribution of taxes between the parties, but the Z value of centralized investments in collective goods will vary. In computer calculations we assumed (in contrast to the previous case) , that the utility function of each participant now depends on all three components of the collective good : *F1* + *F2* + *Z*. Calculations showed that the consumption value x_k, labor supply issue will stabilize over time , but the price variables *p* , *q*, *D* go out on a constant rise , keeping together the same proportion. Interestingly, in model 3 , as in model 1, when the equation $a_k\ \lambda_k = a_i\ \lambda_i$ is fulfill, all equilibrium natural values coincides with optimal performance (using in the function *W* these coefficients λ).

To determine the optimal values of the variables we solve the system of equations:

$$A1 \cdot l1^{\alpha 1} + A2 \cdot l2^{\alpha 2} - a - \left(\frac{\lambda 1}{p} + \frac{\lambda 2}{p} \right) - F = 0$$

$$\frac{\lambda 1 \cdot a1 + \lambda 2 \cdot a2}{F} - p \le 0 = 0$$

$$\frac{-\lambda 1 \cdot b1}{T1 - l1} + p \cdot A1 \cdot \alpha 1 \cdot l1^{\alpha 1 - 1} = 0 \qquad F \ge 0$$

$$\frac{-\lambda 2 \cdot b2}{T2 - l2} + p \cdot A2 \cdot \alpha 2 \cdot l2^{\alpha 2 - 1} = 0 \qquad F \cdot \left(\frac{\lambda 1 \cdot a1}{F} - p \right) = 0$$

The values obtained are presented in Table 6.

Table 5: Parameters of the model 3

a	a_k	b_k	a_k	T_k	α_k	λ_k
35	0.39	0.156	15	16.1	0.9	8
35	0.58	0.23	18	14.6	0.88	5.38

Table 6: Founded indicators model 3 under condition (3)

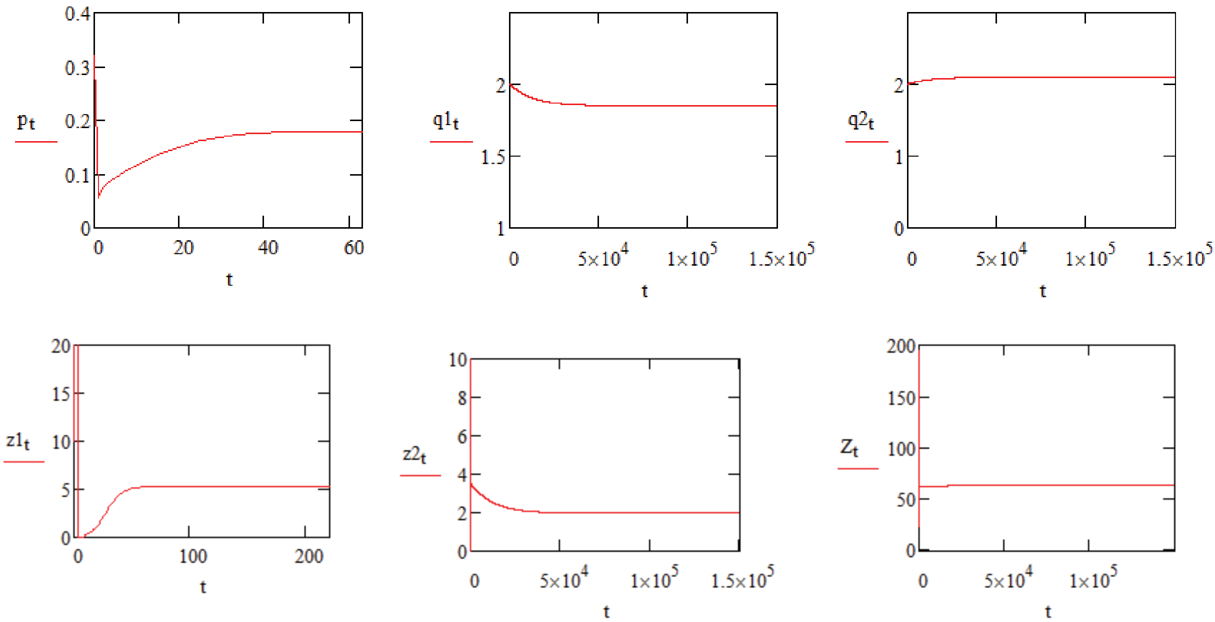
	X1	X2	l1	l2	y1	y2	F1+F2+Z	p	q1	q2	λ1	λ2
Equilibrium	140.79	94.671	13.98	12.34	161.103	164.27	5.5+12+37.4	0.167	1.732	1.957	8	5.38
Optimum	140.79	94.671	13.98	12.34	161.103	164.27	54.91	0.057	0.589	0.667	8	5.38

Table 7: Founded indicators model 3

	X1	X2	l1	l2	y1	y2	F1+F2+Z	p	q1	q2	λ1	λ2
Equilibrium	141.87	93.99	14.38	12.39	161.217	164.97	19.6+0+35.71	0.73	1.8	2.03	8	5.3
Optimum	141.54	93.77	13.97	12.36	160.99	164.52	55.199	0.06	0.6	0.66	8	5.3

Price ratio 3.068 and the following graphs for Table 6.

Figure 3: The trajectories of convergence indicators to equilibrium model 3



4. CONCLUSION

With the advent of computer technology opportunities analysis of complex socio-economic objects have increased dramatically, not only due to the growth rate of statistical information processing, but also due to the possibilities of the qualitative analysis of the virtual world, reality counterparts. In spite of the greater complexity of the objects being studied, in order to understand some of the most common properties reality, we often can and should reflect these properties in the form of simple models.

We used a single-product economic model with two participants and the most simple expressions for their utility functions - in the form of Cobb-Douglas function. (Note that in many other models our methods work equally well for other functions, such as functions of CES).

In the models considered above we clearly distinguished role of the Centre (the state) and market participants - industries and households. Households own labor involved in manufacturing the product, salary spend on consumption and the creation of a collective good. Center always asks one or another method of distribution of the tax burden, and may also generate the value of its contribution to the collective good.

Note that although the policy of the Centre depends on the nature of equilibrium, the market equilibrium can not increase the value of the social welfare function, when the parameters of this function are fixed. It seems that using aggregate models, for example, any regional economies can be compared with market-based options-optimal planning, but it needs to be based on the real statistical data.

Original procedure that ensures social harmony without justifiable public good, we "recommend" to the Centre for Tax Policy and formation in the presence of the collective good. As shown in the simplest computable models, this procedure provides a stable equilibrium, but it does not guarantee optimality. And although there are publications [3], in which presents theorems on Pareto optimality of equilibrium with public goods, yet they belong rather to other variants of models which consider additional conditions. Therefore, in our opinion, in the formulation of social and economic policy must always be considered and consolidated options and optimal equilibrium models.

We add that the scheme given in Annexes for finding equilibrium and simple program for calculating and constructing trajectories (here - in a package MATHCAD) can easily reproduce by any reader to view the dynamics of processes for different values.

Appendix 1.

Consider the classical problem for using collective good for the two parties.

Let the utility function of both players are expressed as follows:

$$u_k = \ln x_k + a_k \cdot \ln(F_1 + F_2), \qquad (a)$$

with constraints:

$x_k+F_k \le w$,

$$x_k + F_k \le w, \quad x_k,\ F_k \ge 0, \quad k=1,2... \qquad (b)$$

If a Nash point exists, it satisfies the conditions maximizing u_k under the above restrictions. Lagrange conditions of equality implies:

$$F_1 = \frac{w_1 \cdot a_1 - F_2}{1 + a_1},$$

$$F_2 = \frac{w_2 \cdot a_2 - F_2}{1 + a_2}$$

from where it is easy to find its values.

Equilibrium point of this game can be computed (in our case) using the sequential procedure to improve their condition by each participant (x_k^*, F_k^*), if he knows the aggregate remaining steps in the previous time *t-1*.

That is at every step t +1 decides the conditions of utility maximization:

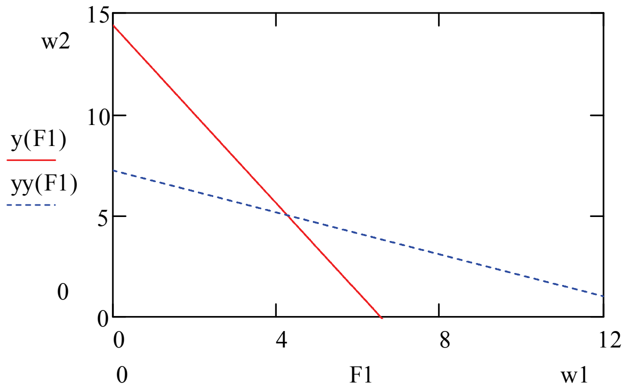
$$(F_k)^{t+1} = \arg\max u_k (w_k - F_k, \sum_{i \ne k} F_i^t), \quad 0 \le F_k \le w_k.$$

For logarithmic utility functions and for given values of the parameters of the game we get a convergent sequence of solutions shown in Figure.

Let the parameters of game set

w1=12, w2=15, a1=1.2, a2=0.93

$$y(F1) = -(1 + a1) \cdot F1 + a1 \cdot w1, \qquad yy(F1) = \frac{-1}{1 + a2} \cdot F1 + \frac{a2 \cdot w2}{1 + a2}$$



$$F := \begin{pmatrix} 4.264 \\ 5.018 \end{pmatrix}$$

In order that this solution is Pareto-optimal, it is necessary the existence of positive λ-coefficients, for which the function $w=\lambda 1u1+\lambda 2u2$ has peaks in the specified rectangle. Conditions Lagrange prescribed for variables $F1 \neq F2 \neq 0$ and 0, have the form:

$$\frac{\lambda 1}{w1-F1}=\frac{\lambda 1\cdot a1+\lambda 2\cdot a2}{F1+F2} \qquad \frac{\lambda 2}{w2-F2}=\frac{\lambda 1\cdot a1+\lambda 2\cdot a2}{F1+F2}$$

Substituting in these relations founded values for F1 and F2, we get a homogeneous linear system with respect to *λ1, λ2*. The existence of nonzero *λ* requires in order that determined *D* of the homogeneous system (conditions of Lagrange) would be equal to zero, but it is not so:

$$D := \begin{bmatrix} 9.283 + a1\cdot\left(w1 - F_0\right) & -a2\cdot\left(w1 - F_0\right) \\ -a1\cdot\left(w2 - F_1\right) & 9.283 + a2\cdot\left(w2 - F_1\right) \end{bmatrix} \qquad |D|=258.51$$

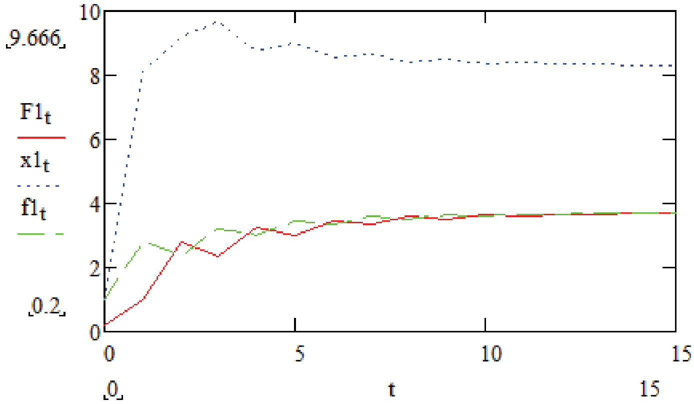
Hence, the value F cannot be optimal. For other values of the parameters we find the path to enter equilibrium.

a1=0.52, a2=0.3, w1=12, w2=15
$F1_0=0.2$, $F2_0=0.3$, $f1_0=1$, $f2_0=2$, $x1_0=1$, $x2_0=1$

Computing solutions

$t := 0..50$

$$\begin{pmatrix} x1_{t+1} \\ x2_{t+1} \\ F1_{t+1} \\ F2_{t+1} \\ f1_{t+1} \\ f2_{t+1} \end{pmatrix} := \begin{pmatrix} \min\left(w1, \dfrac{F2_t + w1}{a1+1}\right) \\ \min\left(w2, \dfrac{F1_t + w2}{a2+1}\right) \\ f1_t \\ f2_t \\ \max\left(0, \dfrac{w1\cdot a1 - f2_t}{1+a1}\right) \\ \max\left(0, \dfrac{w2\cdot a2 - f1_t}{1+a2}\right) \end{pmatrix}$$



The figure shows that the convergence achieving of equilibrium fast.

Appendix 2.

Software calculations provide:

1. Solution of the linear equation to determine the taxation of households to ensure performing equations (3).
2. Iterative procedure to determine the trajectories of prices and private investments in the collective good .
3. Calculating the values of the economy indicators.
4. Plotting trajectories.

The size of step h, the number of iterations and the initial values of the variables are determined by researcher depending on the model parameters and capabilities of the computer Further we present program for calculating equilibrium paths for model 3. In this case, the number of steps N = 150000 .

$$h := \begin{pmatrix} 0.00001 & 0 & 0 & 0 & 0 \\ 0 & 0.000016 & 0 & 0 & 0 \\ 0 & 0 & 0.000011 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The initial values of the variables:

$P_0=0.101$, $q1_0=1.97$, $q2_0=1.8$, $z1_0=10$, $z2_0=22$, $Z_0=40$.

Calculations for a model with two producers and two households in the package MATHCAD, executes with consistent procedures. At each time point t depending on the prices p, q1, q2 are calculated - the optimal demand for labor on the part of producers and their corresponding issues y1, y2. Household behavior is focused on the assumption that the other party execute e an investment in a collective good , as the average of the previous two - will be chosen (z1, z2). Next, we solve the problem of the Centre for allocation of taxes under the assumption that the total income will be the same. Algebraic system is solved

D1+D2=D and $\lambda_k \cdot \beta_k(D_k) = \lambda_i \cdot \beta_i(D_i)$

After this price variables and investments in collective benefit determined by the system difference equations. Investment of Center Z computes by a single formula on the basis of the current price.

$$R := \text{for } t \in 1..N$$

$$l1_{t-1} \leftarrow \left(\frac{q1_{t-1}}{p_{t-1}\cdot A1\cdot \alpha 1}\right)^{\frac{1}{\alpha 1-1}}$$

$$l2_{t-1} \leftarrow \left(\frac{q2_{t-1}}{p_{t-1}\cdot A2\cdot \alpha 2}\right)^{\frac{1}{\alpha 2-1}}$$

$$y1_{t-1} \leftarrow A1\cdot\left(l1_{t-1}\right)^{\alpha 1}$$

$$y2_{t-1} \leftarrow A2\cdot\left(l2_{t-1}\right)^{\alpha 2}$$

$$\Pi_{t-1} \leftarrow p_{t-1}\cdot y1_{t-1} - q1_{t-1}\cdot l1_{t-1} + p_{t-1}\cdot y2_{t-1} - q2_{t-1}\cdot l2_{t-1}$$

$$\text{if } t \geq 5$$

$$z1_{t-1} \leftarrow \frac{z1_{t-1} + z1_{t-2}}{2}$$

$$z2_{t-1} \leftarrow \frac{z2_{t-1} + z2_{t-2}}{2}$$

$$D2_{t-1} \leftarrow \frac{1}{1+B}\cdot\left[p_{t-1}\cdot a - \Pi_{t-1} + B\cdot\left[q2_{t-1}\cdot T2 + p_{t-1}\cdot\left(Z_{t-1} + z1_{t-1}\right)\right] - q1_{t-1}\cdot T1 - p_{t-1}\cdot z2_{t-1}\right]$$

$$D1_{t-1} \leftarrow p_{t-1}\cdot a - \Pi_{t-1} + p_{t-1}\cdot Z_{t-1} - D2_{t-1}$$

$$x1_{t-1} \leftarrow \frac{T1\cdot q1_{t-1} - D1_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z2_{t-1}\right)}{p_{t-1}\cdot(1 + a1 + b1)}$$

$$x2_{t-1} \leftarrow \frac{T2\cdot q2_{t-1} - D2_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z1_{t-1}\right)}{p_{t-1}\cdot(1 + a2 + b2)}$$

$$\begin{pmatrix} p_t \\ q1_t \\ q2_t \\ z1_t \\ z2_t \end{pmatrix} \leftarrow \begin{pmatrix} p_{t-1} \\ q1_{t-1} \\ q2_{t-1} \\ 0 \\ 0 \end{pmatrix} + h\cdot\begin{bmatrix} x1_{t-1} + x2_{t-1} - y1_{t-1} - y2_{t-1} + z1_{t-1} + z2_{t-1} + a + Z_{t-1} \\ l1_{t-1} - \left[T1 - \dfrac{b1\cdot\left[T1\cdot q1_{t-1} - D1_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z2_{t-1}\right)\right]}{q1_{t-1}\cdot(1 + a1 + b1)}\right] \\ l2_{t-1} - \left[T2 - \dfrac{b2\cdot\left[T2\cdot q2_{t-1} - D2_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z1_{t-1}\right)\right]}{q2_{t-1}\cdot(1 + a2 + b2)}\right] \\ \max\left[\dfrac{a1\cdot\left[T1\cdot q1_{t-1} - D1_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z2_{t-1}\right)\right]}{p_{t-1}\cdot(1 + a1 + b1)} - Z_{t-1} - z2_{t-1}, 0\right] \\ \max\left[\dfrac{a2\cdot\left[T2\cdot q2_{t-1} - D2_{t-1} + p_{t-1}\cdot\left(Z_{t-1} + z1_{t-1}\right)\right]}{p_{t-1}\cdot(1 + a2 + b2)} - Z_{t-1} - z1_{t-1}, 0\right] \end{bmatrix}$$

$$Z_t \leftarrow \frac{(\lambda 1\cdot a1 + \lambda 2\cdot a2)}{p_{t-1}}$$

$$v_0 \leftarrow p$$

$$v_1 \leftarrow q1$$

$$v_2 \leftarrow q2$$

$$v_3 \leftarrow z1$$

$$v_4 \leftarrow z2$$

$$v_5 \leftarrow D1$$

$$v_6 \leftarrow D2$$

$$v_7 \leftarrow Z$$

$$v$$

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