CAPITAL AUGMENTING AND LABOR AUGMENTING APPROACH IN MEASURING CONTRIBUTION OF HUMAN CAPITAL AND EDUCATION TO ECONOMIC GROWTH

KAPITAL UVEĆAVAJUĆI I RAD UVEĆAVAJUĆI PRISTUP U MJERENJU DOPRINOSA LJUDSKOG KAPITALA I OBRAZOVANJA EKONOMSKOM RASTU

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Abstract: *In this paper an effort has been made to unveil some hidden and implicit assumptions that has been used in different models dealing with analysis and measurement of contribution of human capital to economic growth. We start from general production function with heterogeneous labor input and general production function with heterogeneous human and physical capital. By introducing different assumptions regarding partial elasticity of substitution between different factors of production we derived different models for human capital contribution. Apart from making hidden assumptions of existing models explicit, we also derived dozen of others models that can be used for same purposes. Among those newly proposed models especially important are those that are derived from general production function with heterogeneous human capital and that are based on assumption of unlimited partial elasticity of substitution between different kinds of human capital. First, they allow for more detailed sources of growth analysis. Second, they do not have any problem with wage premium increase experienced in last three decade, which make problematic usage of most of other models used so far.*
Key words: *Economic Growth, Human Capital, Education, Partial Elasticity of Substitution*

Apstrakt: *U ovom člamku je učinjen napor da se otkriju neke sakrivene i implicitne pretpostavke koje su sadržanne u raznim modelima koji se bave analizom i merenjem doprinosa ljudskog kapitala privrednom rastu. U analizi se polazi od opšte proizvodne funkcije sa heterogenim inputom rada i od opšte proizvodne funkcije sa hetrogenim ljudskim i fizičkim kapitalom. Uvodjenjem različitih pretpostavki vezanih za parcijalnu elastičnost supstitucije izmedju različitih faktora proizvodnje uspeli smo da deriviramo niz različitih modela za merenje doprinosa ljudskog kapitala privrednom rastu. Pored toga sto smo implicitne pretpostavke postojećih modela učinili eksplicitnim, derivirali smo i niz drugih modela koji se mogu koristiti za svrhe iste analize. Medju tim novopredloženim modelima posebno su važni oni koji su derivirani iz opšte proizvodne funkcije sa heterogenim ljudskim kapitalom a koji se zasnivaju na pretpostavci da postoji beskonačna elastičnost supstitucije izmedju različitih vrsta ljudskog kapitala. Prvo, oni omogućavaju detaljniju analizu izvora privrednog rasta. Drugo, oni nemaju problema da apsolviraju činjenicu o rastućim premijama na obrazovanje, što predstavlja veliki problem kod modela koji se sada koriste u analizi.*
Ključne reci: *ekonomski rast, ljudski kapital, obrazovanje, parcijalna elasticnost supstitucije*

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1. Introduction: Solow, Capital Augmenting, and Labor Augmenting Approach

Since the late eighties and early nineties Solows' neoclassical model of growth has come under attack because of its' inability to provide *an empirically adequate theory of growth.* One of the most important characteristic of this growth accounting practice is *low level of elasticity of output with respect to capital* (let's note it as *a*), *and consequent strong effect of diminishing return on capital:* value of coefficient *a* usually used in different empirical research was about 1/3. As a consequence, model encounters several difficulties. **First**, this kind of measurements show that increase in capital labor ratio can explain just 10-20% of long run growth rate of per capita output. Remaining 80-90% are left unexplained and are simply termed as technological progress, global (or total, or combined) factor productivity growth, or simply, and probably most appropriately, as *residual.* Unfortunately, critics claim, Solow's theory has nothing to say neither about anatomy of this residual, nor about policy that might influence it. It is especially important shortcoming having in mind magnitude of this residual. **Second** important problem of those measurements refers to the fact that, above mentioned, *property of sharply diminishing return on capital places sharp limit on models ability to explain cross-country differences in per capita income*. Doubling of capital stock per capita will increase steady state of income by just 26% [$(2^{1/3} - 1)\ 100 = 26$]. Obviously, large differences in capital per capita produce small differences in output per capita. It is quite obvious, and it can bee proven more rigorously[2], that this property and behavior of the model crucially rest on the magnitude of **a:** with larger value for **a** than 1/3 more cross-countries variations in the level of income per capita can be explained by variation in capital stock per capita. **Third** property and shortcoming of those measurements is that, owing again to sharp diminishing returns to capital, *model is unable to explain cross-country differences in the rates of growth by just referring to transitory dynamics and country's position on transitory path.* Connected with this is problem of the *length of transition period (*or length of adjustment or convergence period). Model predicts, according to Mankiw (1995), that annual rate of adjustment should be about 4% per year. Statistical data indicate, however, that the rate of adjustment is as large as half of that value, and that convergence period might be about twice of that implied by estimated rate of adjustment. Again, models estimate of the speed of adjustment and length of transitory period crucially rest on the assumption on the magnitude of elasticity of production with respect to capital**.** Increase of **a** by twice, from 1/3 to 2/3, would bring estimated rate of convergence and length of transition period to the magnitudes that are in accordance with real statistical data. Finally, **forth,** empirical difficulty of Solow's model refers to its *inability to explain cross-countries differences in rate of returns on capital*. This model predicts that, in the case of Cobb-Douglas production function, which assume unit elasticity of substitution and *a = 1/3,* typical poor country, which has ten times smaller income per capita than typical rich country, should have about one hundred times larger rate of return to capital than that in rich countries. More specifically, since average rate of return in rich countries is about 10%, model predict that rate of return in poor countries should be about 1000%. It more than anything else contradicts to what we have in reality.[3]

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[2] For rigorous proof and elaborate consideration of this problem in the case of general production function see Mankiw (1995).

[3] As noted result is extremely dependent on value of *a*. But it is also very dependent on the value of elasticity of substitution. Reasonable increase of *a* from 1/3 to 2/3 and increase of elasticity of substitution from 1 to 4, which can be appropriate for large and / or very open countries, can bring predictions of the model to the realistic level. For more details see Mankiw (1995). Importance of elasticity of substitution in explaining cross-country differences is recently stressed in empirical work done by Caselli, F. (2004). Apart from it Caselli point to the importance of differences in capital structure and different behavior of government sector as a possible explanation.

As already stressed, main source of all above mentioned difficulties refers to the fact that capital exhibits extremely strong decreasing marginal productivity, or what is most of the time same thing, strong diminishing returns. An upward correction of value of elasticity of production with respect to capital, that is an increase of **a**, would no doubt significantly reduce gap between predictions of the Solow model and recorded statistical data, and in that way help salvage Solows' neoclassical theory of growth. It is exactly what has been proposed and empirically tested by some authors [Mankiw et al (1992), Mankiw, G. (1995)].[4] In order to do it, first, very *concept of capital* had to be redefined. The understanding and measurement of reproducible capital has been broadened to include all different forms of intangible capital. Most importantly, it was broadened to include so-called *human capital.* Most important part of human capital is *capital of education* (ED capital). So the idea was to include all those investments in education that, by improving the quality of labor, are being embodied in labor force. Capital of education is not only most important part of human and intangible capital, but is also regarded as a good proxy for all other form of intangible / human capital. At the same time it is relatively easy to measure, while other forms of intangibles are not. For that reason we will also refer only to educational capital. Second, concept of elasticity of production with respect to capital is naturally broadened to include both share of conventional capital and share of human / educational capital in gross domestic product.

In order to see differences between Solows' original model and this new model let's take a look at production functions used in generating two models of growth. Sollow (1956, 1957) starts with Cobb-Douglas production function of the following form:

$$Q_t = A_t H_t^b K_t^a \qquad (1)$$

Here H_t stands for labor hours, while K_t stands for conventional (tangible) capital used for production, Q_t, in period t; A_t is level of technical efficiency; a presents, as already said, elasticity of production with respect to conventional capital, while b presents elasticity of production with respect to labor[5].

On the other hand, newly proposed production function has been given in two versions. First one is based on the following form

$$Q_t = A_t H_t^{b_u} K_t^a E_t^f = A_t H_t^{b_u} C_t^{a+f} = A_t H_t^{b_u} C_t^h \qquad (2)$$

Here

$$C_t = K_t^{a/(a+f)} (\sum_{i=1}^{n} E_{it})^{f/(a+f)} = K_t^{a/(a+f)} E_t^{f/(a+f)} \qquad (3)$$

can be treated as a total capital, physical and human, aggregated using geometric index, while

$$E_t = \sum_{i=1}^{n} E_{it} \qquad (4)$$

presents human capital or capital of education measured as the ordinary sum of all kinds of human / educational capital. Coefficient h stands for newly defined elasticity of production with respect to newly defined capital. It is equal to the sum of elasticity of production with respect to conventional capital, a, and elasticity of production with respect to educational capital, f, that is

$$h = a + f \qquad (5)$$

[4] Answer of some other authors on those problems presented radical departure from Solows' neoclassical model and development of what is now known as New Theory of Growth. Cornerstones of this new theory are early articles of Romer, P. M. (1986) and Lucas, R. E. (1988). See also Romer, P. M. (1987, 1990, 1994, 1997) and Lucas, R. E. (1993). For short and simple exposition of recent development of old and new theory see Xaviar Sala-i-Martin (2002).

[5] It is usually assumed that $a+b=1$, meaning that economy of scale coefficient is equal to one, and that therefore $b=1-a$. In this paper, we will follow this tradition but will continue to use more general notation, b instead of $1-a$.

Coefficient b_u presents elasticity of production with respect to “raw” labor, that is with respect to unqualified part of labor force. Here again $a+b_u+f=b_u+h=1$. This form has been proposed and empirically treated in Mankiw et al 1992 article.

Second version of the model has been first time proposed in Mankiws’ 1995 article. Unfortunately it was not formally given in quoted article. Instead, what we have in Mankiws article is following statement:

> *When applying neoclassical model to understand international experience, it seems best to interpret variable **k*** (***K*** in our expression ***(1)***) *as including all kinds of capital. Thus, the capital share, **a**, should include the return to both physical and human capital.* (Mankiw, 1995, page 293).

This, in the case of Cobb-Douglas production function[6], can formally be expressed in the following way

$$Q_t = A_t H_t^{b_u}[E_t + K_t]^{(a+f)} = A_t H_t^{b_u} C_t^h \qquad (6)$$

H_t again presents hours of work, A_t is level of technology, while b_u stands for elasticity of production with respect to “raw” labor. On the other hand, C_t now stands for sum of conventional, K_t, and educational (or human) capital, E_t, given as an ordinary sum of those two kinds of capital, that is

$$C_t = E_t + K_t = \sum_{i=1}^{n} E_{it} + K_t \qquad (7)$$

Educational capital is, as in previous case, supposed to be measured as a simple sum of all kinds of educational capital. Again, h stands for newly defined elasticity of production with respect to newly defined capital. Again, it is equal to the sum of elasticity of production with respect to conventional capital, a, and elasticity of production with respect to educational capital, f.

Having latest couple of expressions in mind we will, for the sake of simplicity, call this approach *capital augmenting* or *capital adjusted approach.* On the first glance differences between two models, ***(1)*** on the one hand and ***(2)*** and ***(6)*** on the other hand, and their underlying assumptions are straightforward and easy to understand. However, if we start from more general *production function with different forms of educational capital* we will see that it is not so, and that there are some silent assumptions in *capital adjusted approach* that are not so obvious at all and that are not spelled out explicitly by its’ advocates. So, first motivation of this article is to make those silent assumptions more explicit and clear. By evaluating reality of those assumptions we will be able to evaluate reality of newly proposed model itself.

On the other hand, concepts of human capital and investment in education are old one. They were, for the first time formally and explicitly, introduced more than four decades ago in the works of Schultz, Backer, Hansen, Mincer, Blaug, and others. Approximately at the same time, education and human capital were introduced in the economic growth theory. Those early as well as later efforts by Denison E., Schultz T., Pasharopoulos G., Kendrick J., Jorgenson D., and Griliches Z. were mainly concerned with contribution of education and human capital to economic growth. They present part of so called *sources of growth analysis,* whose main aim is decomposition and explanation of Solows' residual. All those analysis and measurements show that education and human capital in general present one of the most important source of economic growth, and that “raw” labor is much less important than it might be implied by early studies. Adding contribution of human capital to contribution of conventional, physical capital gives measure for gross

[6] Note that Mankiw in quoted article uses general production function.

contribution of all sorts of investing and saving to economic growth. Obviously, those kinds of growth models show much more sensitivity to overall investment and saving rate than original one.

Contribution of education to economic growth is expressed in all those studies in, more or a less, the same way: total contribution of labor force is decomposed into part that measure contribution of "raw" labor (unskilled part of all workers) and total contribution of education (skilled part of all workers). More formally, this result is usually obtained using a sort of production function similar to one given in expression (***1***), except that labor input is measured, not by number of employee or their hours of works, but by quality or efficiency adjusted labor index. This index is almost always calculated as a weighted sum of quantity of different kind of labor (education), where relative level of wages and salaries of those kinds of labors (education) are used as weights. Formally

$$Q_t = A_t K_t^a H_t^{*b} \qquad \textbf{\textit{(8)}}$$

where H_t^* stands for efficiency adjusted labor input while A_t again stands for level of technological efficiency[7]. Having this in mind we will, for the sake of simplicity, from now on call this method *labor augmenting* or *labor adjusted approach.*

It is now possible to show, and it is second motivation for this article, that differences between *labor input adjusted approach* and new *capital adjusted approach* are not substantial as it might seem at the beginning, and that those two approaches, in fact, belong to the same school of thinking. To see it note that both kind of those two factor production functions - labor adjusted and capital adjusted - can be obtained using more general multi factor production function which is equipped, apart from conventional capital and labor inputs, with inputs of different kind of labor (education) or with inputs of different kind of educational capital. Taking now specific assumption with respect to behavior of partial elasticity of substitution between different factors will produce those two different kinds of two-factor production function. This undertaking will consequently show that, although two approaches belong to the same school of thinking, they do not necessarily produce same empirical results and predictions owing exactly to different assumptions about partial elasticity of substitution they are based upon. This comparative analysis of old, labor adjusted, and new, capital adjusted, approach is very important especially in the light of the fact that both approach exhibit strong sensitivity of GDP rate of growth with respect to rate of investment in all forms of capital. It is striking that this similarity has never been properly explored.

Empirical context in which new, capital adjusted approach and labor input adjusted approach have been used is, however, very different. Old labor input adjusted approach has been mostly used in an effort to decompose rate of growth (dynamic analysis), while new approach has been mostly used in order to explain cross-countries differences in the level of development (comparative static analysis) and differences in the rate of growth (comparative dynamic analysis). It is obvious, however, that labor-adjusted approach can be, equally legitimately and with similar (but not same) results, used to explain cross-country differences. Similarly, capital adjusted approach can be used for measurement of contribution of different kind of education to economic growth. In other words, theory which is able to explain differences in productivity between two points in time (sources of growth analysis) can equally legitimately be used to explain differences in productivity between two points in space (cross-country analysis) and vice versa. Having this in

[7] It is very important to note that magnitudes of A_t's from expressions (1), (2), (6) and (8) are not the same. Reason for it is quite obvious: factors of production are aggregated in these expressions in different ways. Same apply for A_t's and its rates of growth ($\dot{A}/A$) in different models and expressions that follow in the rest of this paper. It is only for the sake of simplicity that we use same notation for the level of technological efficiency (A_t) and rate of growth of global factor productivity ($\dot{A}/A$) in all those different models.

mind, in what follow we will, for the sake of simplicity and to avoid unnecessary repetition, outline only dynamic analysis framework.

In the rest of the paper we will develop two more general approaches that will help us to fulfill above-mentioned aims of this article. First approach is one that starts from general multifactor production function with different kinds of labor as inputs. Differences in education of labor force are here used as most important. Taking now different restrictions about different parameters of this general production function we will arrive at different forms of labor input adjusted two-factor production functions. This approach will be presented in next section. Second approach start from the general multifactor production function with different kind of human and educational capital. Again, taking different restrictions about different parameters of this general production function we will arrive at different forms of capital adjusted two-factor production functions. This approach will be presented in third section. Main conclusions of analysis are presented in final section.

2. Models with Heterogeneous Labor Inputs

1. In order to understand underlying assumptions of labor input adjusted approach in analysis of contribution of education to economic growth, we will start from general multi factor production function of the form[8]

$$Q_t = F\left(K_t, H_{0t}, H_{1t}, H_{2t} ... H_{it} ... H_{nt}, t\right) \qquad \textbf{(9)}$$

where K_t stands for capital, H_{it} for hours of works of *i*-th kind of labor (those with *i* years of schooling) and *t* presents time. By differentiating and dividing with Q_t we are getting rate of growth of production (GDP, in the case of aggregate economy) decomposed in the following way

$$\dot{Q} / Q = \dot{A} / A + a_t\left(\dot{K} / K\right) + \sum_{i=0}^{n} b_{it}\left(\dot{H}_i / H_i\right) \qquad \textbf{(10)}$$

Coefficient $a_t = (F_{Kt} K_t / Q_t)$ presents elasticity of production with respect to capital, while $b_{it} = (F_{Hit} H_{it} / Q_t)$ presents elasticity of production with respect to *i*-th kind of labor. As usual, $F_{Kt} = \partial Q/\partial K$ stands for marginal productivity of capital and $F_{Hit} = \partial Q/\partial H_i$ stands for marginal productivity of *i*-th type of labor. Obviously, first element, $\dot{A} / A$, presents contribution of global factor productivity to growth rate of GDP, second element, $a_t(\dot{K} / K)$, measures contribution of capital accumulation, while last element, $\sum b_{it}(\dot{H}_i / H_i)$, express contribution of all types of labor to the rate of growth[9].

[8] Note that results that follow could have been derived also if we started from general production function loaded with quality adjusted labor input, H^*, which is itself function of different kind of labor, $H^* = h(H_0, H_1, H_2, ...H_i...H_n)$. This procedure has been for example used by Mulligan and Sala-i-Martin (1995b). For a simple exposition see also Stevens, P. and Weale, M. (2003). This approach seems to be less general than one proposed here because it starts at the very beginning with assumption that partial elasticity of substitution between different kinds of labor is independent of quantity of physical capital and that it depends only on quantity of different kinds of labor. Another interesting approach in measuring human capital, proposed by same authors (Mulligan and Sala-i-Martin, 1995a), is based on an effort to find optimal index number, that is the index number that minimize a function of the expected error made when human capital indexes are compared across different economies.

[9] Influence of education and human capital on economic growth is here somewhat simplified. It is assumed that education has only direct influence on output. However, its indirect impact, via rate of creation and diffusion of new technologies and new knowledge in general, as elaborated by Nelson R. and Phelps, E. (1996), can be even more important than this direct impact. The new growth theory, relying on the idea of educational externalities, also emphasizes the higher rate of innovation that can be generated by having more educated workers generating new ideas. For a survey of those theories and concepts see Sianesi, B. and Van Reenen, J. (2000, 2002). See also Dowrick, S. (2002).

Last element is in fact the sum of contributions of all types of labor educational categories to economic growth. It can be further transformed to give

$$\sum_{i=0}^{n} b_{it}(\dot{H}_i / H_i) = b_t \sum_{i=0}^{n} m_{it}(\dot{H}_i / H_i) \qquad \textbf{(11)}$$

where

$$b_t = \sum_{i=0}^{n} \frac{F_{H_{it}} H_{it}}{Q_t} = \frac{H_t}{Q_t} \sum_{i=0}^{n} F_{H_{it}} \left(\frac{H_{it}}{H_t} \right) = \frac{F_{H_t} H_t}{Q_t} \qquad \textbf{(12)}$$

can be interpreted as elasticity of production with respect to aggregate labor, $\sum_{i=0}^{n} F_{H_{it}} \left(\frac{H_{it}}{H_t} \right)$ as marginal productivity of aggregate labor, while

$$m_{it} = \frac{F_{H_{it}} H_{it}}{\sum_{i=0}^{n} F_{H_{it}} H_{it}} = \frac{F_{H_{it}} H_{it}}{F_{H_t} H_t} \qquad \textbf{(13)}$$

can be described as elasticity of aggregate labor share in income with respect to particular kind of labor. Substituting now expression ***(11)*** in ***(10)*** we arrive at

$$\dot{Q} / Q = \dot{A} / A + a_t(\dot{K} / K) + b_t \sum_{i=0}^{n} m_{it}(\dot{H}_i / H_i) \qquad \textbf{(14)}$$

2.1. If we now assume that partial elasticity of substitution between any pars of particular production factors is equal to one and independent of quantity of other factors, it will allow us to simplify initial production function. More specifically, it means that, first, elasticity of substitution between capital (K_t) and any kind of labor (H_{it} for any i) is equal to one and independent of quantities of other kinds of labor. In other words, any change of marginal rate of substitution between capital and particular kind of labor (F_K / F_{Hi} for any i) is followed with change of specific capital labor ratio (K / H_i) of a same proportion. Second, elasticity of substitution between different kinds of labor is equal to one and independent of quantity of capital: any change in marginal rate of substitution between different kinds of labor (F_{Hi} / F_{Hj} for any i and j) is followed with proportional change in ratio of those kinds of labor (H_i / H_j for any i and j). Consequence of those two assumptions is constancy of factors elasticity of production, that is constancy of elasticity of production with respect to capital, $a_t = a$, and constancy of elasticity of production with respect to any kind of labor, $b_{it} = b_i$. Of course, last statement implies constancy of b_t (=b) and m_{it} (=m_i).

Consequently ***(10)*** and ***(14)*** now becomes

$$\dot{Q} / Q = \dot{A} / A + a(\dot{K} / K) + \sum_{i=0}^{n} b_i(\dot{H}_i / H_i) \qquad \textbf{(15)}$$

$$\dot{Q} / Q = \dot{A} / A + a(\dot{K} / K) + b\sum_{i=0}^{n} m_i(\dot{H}_i / H_i) \qquad \textbf{(16)}$$

Now by solving any of those two differential equations (integrating and taking antilogarithm) we get production function of the form

$$Q_t = A_t K_t^a \prod_{i=0}^{n} H_{it}^{b_i} = A_t K_t^a (\prod_{i=0}^{n} H_{it}^{m_i})^b = A_t K_t^a H_t^{*b} \qquad \textbf{(17)}$$

Here, obviously,

$$H_t^* = \prod_{i=0}^{n} H_{it}^{m_i} \qquad (18)$$

presents labor input measured in efficiency-adjusted hours units. Further transformation can be made by multiplying and dividing expression ***(17)*** with H_t^{bu}, where $b_u = F_{H_{0t}} H_t / Q_t$, presents elasticity of production with respect to uneducated part of labor force ("raw" labor),

$$Q_t = A_t H_t^{b_u} K_t^a (\prod_{i=0}^{n} \frac{H_{it}^{m_i}}{H_t^{b_u/b}})^b \qquad (19)$$

Now H_t^{bu} presents input of unqualified labor - "raw" labor, while part in bracket, $(\prod H_{it}^{m_i} / H_t^{b_u/b})^b$, measures contribution of educational input. Obviously third and forth part of this expression together, $K_t^a (\prod H_{it}^{m_i} / H_t^{b_u/b})^b$, measure influence of overall capital (conventional and educational) on economic development. This influence of capital is, obviously, much larger than in original Solows' model.

In this case we are totally in the realm of Cobb-Douglas (CD) production function: all factors of production are aggregated like in CD production function. It is multifactor CD production function. In other words all factors are aggregated using geometric index with factors shares in national GDP as weight. Although very handy this kind of production function has never been used in empirical works dealing with contribution of education to economic growth. Reason is very obvious. While constancy of capital and aggregate labor share in GDP can be regarded as realistic, it is contrary to the very widespread facts to assume constancy of share of different kinds of labor in national product: share of educated categories has increased in last several decades as a result of technological progress and increased demand for educated categories of workers.

2.2. More appealing are assumptions that have been made by Denison and other of the same tradition[10]. First, they assume, implicitly or explicitly, that marginal rate of substitution between different types of labor (F_{Hi} / F_{Hj} for any i and j) does not depend on specific capital labor ratio (K/H_i for any i). This is known as condition of additive separability. It is important because it allows us to solve differential equation ***(10)*** or ***(14)*** by solving separately each part of those equations. Second assumption is that elasticity of substitution between any kinds of labor is unlimited and independent of quantity of any other kind of labor. In other words, changes in ratio of any two kinds of labor (H_{it} / H_{jt} for any i and j) do not have any influence on marginal rate of substitution between those two kinds of labor (F_{Hi} / F_{Hj} for any i and j): Corresponding marginal rate of substitution, F_{Hi} / F_{Hj}, is constant.[11] Taking $j=0$ we can write[12]

$$\frac{F_{H_{it}}}{F_{H_{0t}}} = n_{it} = n_i \qquad (20)$$

[10] See for example works of Denison, E. F. (1962, 1967, 1974, 1985), Kendrick, J. W. (1956, 1979, 1980, 1981), Griliches, Z. (1960, 1963a), Jorgenson D. and Griliches, Z. (1967). See also Madison, A. (1972, 1987), Psacharopoulos, G. (1972). Good survey of concepts can be found in Griliches, Z. (1996).

[11] Note, however, that constancy of F_{Hi} / F_{Hj} can be explained not only with unlimited elasticity of substitution between different kind of labor but also with non-neutrality of technological progress: diminishing returns on investment in education can be compensated with educationally biased technological progress.

[12] Obviously, we can use any other j as numerator. For detailed discussion about reasons for usage of $j=0$ see Mulligan and Sala-i-Martin (1995b). Basically, usage of marginal productivity (wage) of unskilled worker, F_{H0}, is most natural because "zero-schooling person is the same, always and everywhere" while "people with any positive amount of schooling will necessarily be different and, therefore, cannot be used as numeraire".

Finally, third assumption is that elasticity of substitution between capital and aggregate labor is equal to one. Consequence is constancy of elasticity of production with respect to capital (a_t) and aggregate labor (b_t). More formally: $a_t = a$ and $b_t = b$.

By dividing numerator and denominator of expression ***(13)*** with F_{H0} and substituting it in expression ***(14)*** it can be, having in mind previously given assumptions, transformed to give

$$\dot{Q}/Q = \dot{A}/A + a(\dot{K}/K) + b\sum_{i=0}^{n}\frac{(F_{H_{it}}/F_{H_{0t}})H_{it}}{\sum_{i=0}^{n}(F_{H_{it}}/F_{H_{0t}})H_{it}}(\dot{H}_i/H_i)$$

$$\dot{Q}/Q = \dot{A}/A + a(\dot{K}/K) + b\sum_{i=0}^{n}\frac{n_iH_{it}}{\sum_{i=0}^{n}n_iH_{it}}(\dot{H}_i/H_i) \tag{21}$$

Now, by solving this differential equation (again, by integrating and taking antilogarithm) we get specific production function of the form

$$Q_t = A_tK_t^a(\sum_{i=0}^{n}n_iH_{it})^b = A_tK_t^aH_t^{*b} \tag{22}$$

where

$$H^* = \sum_{i=0}^{n}n_iH_{it} \tag{23}$$

presents labor input measured in efficiency-adjusted hours units. More specifically, labor input is here presented in efficiency units of unskilled part of labor force.

Note that relative level of marginal productivity (wages) of different level of education, n_i, following Mincerian tradition, can be expressed as semi-logarithmic function of years of education, i, in which case expression ***(23)*** transforms in

$$H^* = \sum_{i=0}^{n}e^{\phi i}H_{it} \tag{24}$$

Here coefficient ϕ measures influence of level of education (years of schooling) on the relative level of wages (marginal productivity of different kind of labor). In other words relative level of wages (marginal productivity of labor) are here presented as semi-logarithmic function of years of education, $n_i=e^{\phi i}$. For $i=0$ value of $e^{\phi i}$ will, naturally, be equal to 1. This can be further simplified and approximated with

$$H^* = e^{\phi y}H_t = \varphi H_t \tag{25}$$

where $y = \sum i(H_i/H)$ presents average years of schooling in respected economy and where $\varphi=e^{\phi y}$ can be interpreted as human (educational) capital per person employed. Substituting this in ***(22)*** we get

$$Q_t = A_tK_t^a(\varphi H_t)^b = A_tK_t^aH_t^{*b} \tag{26}$$

This is exactly production function proposed by Hall and Jones (1999) and used recently in different cross-country analysis.[13] Obviously this production function is identical in its' nature to one proposed and extensively used by Denison and others more than 45 years ago.

Expression ***(22)*** can be further transformed in

[13] See Caselli (2004) and Jones, C. I. (1996, 2004).

$$Q_t = A_t H_t^{b_u} K_t^a [\sum_{i=0}^{n} \frac{n_i H_{it}}{H_t^{\frac{b_u}{b}}}]^b = A_t H_t^{b_u} K_t^a (H_t^* / H_t^{\frac{b_u}{b}})^b \qquad \textbf{(27)}$$

Obviously, third and forth part of this expression, $K_t^a (H_t^* / H_t^{\frac{b_u}{b}})^b$, present impact of overall capital (conventional and educational) on economic development.

As we see, aggregate labor and capital are here combined using CD production function or by geometric index with share of capital and labor in GDP as weight. On the other hand, labor input is, in this case, aggregated using arithmetic index with fixed marginal rate of substitution between *i*-th kind of labor and unskilled labor (*0*) as weight. Those weights are usually calculated as ratio of wages of different kinds of labor and wages of unskilled labors. This ratio is here assumed to be constant and this is crucial assumption. Although more realistic than assumptions of previous model, and probably realistic and acceptable at the time when it was used by Denison and other, now days it seems pretty unrealistic to assume constancy of so-called wage premium ratio. What we have witnessed in last three decade is steady and significant increase of wage premium ratio. This fact is widely documented by data for most of developed nations. As a matter of fact, this increase of wage premium is one of the most interesting issues in current economic researches, and it still can be regarded as an unsolved puzzle.

2.3. If we in the next step transform equation ***(14)*** in the following form $\dot{Q}/Q = \dot{A}/A + a_t(\dot{K}/K) + b_{0t}(\dot{H}_0/H_0) + (b_t - b_{0t})\sum_{i=1}^{n} m_{it}(\dot{H}_i/H_i)$ and than, after some additional manipulation similar to one from previous paragraph, divide numerator and denominator of its' second, third and forth part with F_{H_0}, we get following decomposition of the rate of growth

$$\dot{Q}/Q = \dot{A}/A + (a_t + b_{0t})\{\frac{(F_{Kt}/F_{H_0t})K_t}{(F_{Kt}/F_{H_0t})K_t + (F_{H_0t}/F_{H_0t})H_{0t}}(\dot{K}/K) + \frac{(F_{H_0t}/F_{H_0t})H_{0t}}{(F_{Kt}/F_{H_0t})K_t + (F_{H_0t}/F_{H_0t})H_{0t}}(\dot{H}_0/H_0)\} +$$
$$(b_t - b_{0t})\sum_{i=1}^{n}\frac{n_{it}(\dot{H}_i/H_i)}{\sum_{i=1}^{n} n_{it}(\dot{H}_i/H_i)}$$

Assume now that all relevant parameters are constant, that is $(a_t + b_{0t}) = (a + b_0)$, $(b_t - b_{0t}) = (b - b_0)$, $(F_{H_it}/F_{H_0t}) = n_{it} = n_i$, and $(F_{Kt}/F_{H_0t}) = n_{Kt} = n_K$. Meaning of these assumptions for behavior of particular partial elasticity is obvious. If we now solve this differential equation we get

$$Q_t = A_t(n_K K_t + H_{0t})^{(a+b_0)}\left(\sum_{i=1}^{n} n_i H_{it}\right)^{(b-b_0)} = A_t(n_K K_t + H_{0t})^{(a+b_0)} H_t^{*(b-b_0)} \qquad \textbf{(28)}$$

where $H_t^* = \sum_{i=1}^{n} n_i H_{it}$ and presents efficiency adjusted labor force of those with any level of education.

If we now assume only two types of labor, unskilled (H_{0t}) and skilled (H_{1t}), and further assume that $n_K = (F_{Kt}/F_{H_0t}) = 1$, which is really very dubious assumption, what we get is

$$Q_t = A_t(K_t + H_{0t})^{(a+b_0)} H_{1t}^{b_1} = A_t(K_t + H_{0t})^{\theta} H_{1t}^{(1-\theta)} \qquad \textbf{(29)}$$

where $\theta = (a + b_0)$ and $1-\theta = b_1$. What we got is obviously noting but celebrated Griliches (1969) expression that implies capital-skill complementarity and that has been used so often in last decade to explain rising wage premium.

2.4. Another, more commonly used way that can capture the idea of capital-skill complementarity and that has a power to explain rising wage premium is so called CES approach proposed and tested by Krusell at all (1997). Assuming constant partial elasticity of substitution between different factors of production and assuming only 3 factors of production (capital, *K*, skilled labor, H_1, and unskilled labor, H_0)[14], by solving adequate differential equation we can derive nested CES production function of the following form

$$Q_t = A_t\left(\mu H_{0t}^{\sigma} + (1-\mu)\left(\lambda K_t^{\rho} + (1-\lambda)H_{1t}^{\rho}\right)^{\frac{\sigma}{\rho}}\right)^{\frac{1}{\sigma}} \qquad \textbf{\textit{(30)}}$$

2.5. If we now in equation ***(14),*** for the sake of simplicity substitute $H_{(n+1)t}$ for K_t and $n_{(n+1)t}$ for $n_{Kt} = F_{Kt} / F_{H0}$, and than make certain simple manipulation, than knowing that $(a_t + b_t) = 1$, we get following decomposition of the rate of growth $\dot{Q}/Q = \dot{A}/A + (a_t + b_t)\sum_{i=0}^{n+1} m_{it}(\dot{H}_i / H_i) = \dot{A}/A + \sum_{i=0}^{n+1} m_{it}(\dot{H}_i / H_i)$. Further, if we divide numerator and denominator of this m_{it} with $F_{H_0 t}$, than assume constancy of parameter n_{it}, that is $n_{it} = \dfrac{F_{Hit}}{F_{H0t}} = n_i$, and solve this differential equation we get

$$Q_t = A_t(\sum_{i=0}^{n+1} n_i H_{it}) = A_t(n_K K_t + H_{0t} + \sum_{i=1}^{n} n_i H_{it}) = A_t[H_{0t} + (n_K K_t + H_t^*)] \qquad \textbf{\textit{(31)}}$$

This is obviously linear production function. If we now drop out part H_{0t} from this equation but still keep assumption of constant economy of scale we get

$$Q_t = A_t(n_K K_t + H^*) = A\overline{K} \qquad \textbf{\textit{(32)}}$$

which in fact, by its nature, presents well-known "AK" model, where capital is expressed in efficiency units of unskilled labor.

Assuming, on the other hand, only one type of labor (average labor) in expression ***(31)***, and assuming equality of marginal products and factors prices, expression ***(31)*** becomes

$$Q_t = A_t[n_K K_t + \sum_{i=0}^{n} n_i (H_{it} / H_t) H_t] = A_t[n_K K_t + nH_t] = A_t[(\pi / w_0)K_t + (w / w_0)H_t] \qquad \textbf{\textit{(33)}}$$

where π presents price of capital, w_o wages of unskilled workers, while w stands for average wage. This is similar to well known linear production function used by Abramovitz, M. (1956) in one of the first sources of growth analysis. Note, however, that in this expression we use relative level of factor prices, while Abramowitz uses absolute level of factor prices, that is

$$Q_t = A_t[\pi K_t + wH_t] \qquad \textbf{\textit{(33')}}$$

3. For the purpose of further analysis and in order to give full survey of this approach we will now decompose the rate of growth in a bit more detailed way. If we add and subtract, in expression ***(11)***, ordinarily measured contribution of homogenous labor to economic growth,

[14] In their original work they use 4 factors of production by distinguishing between capital structures from capital equipment. Also they introduced labors efficiency adjustment factors, something we will introduce latter in this work.

$b_t(\dot{H}/H)$, it will not change value but will allow us to decompose total labor contribution in more detailed way

$$\sum_{i=0}^{n} b_{it}\frac{\dot{H}_i}{H_i} \approx b_t\frac{\dot{H}}{H} + b_t\sum_{i=0}^{n}(\frac{F_{H_{it}}}{F_{H_t}})\Delta(H_i/H) \qquad \textbf{\textit{(34)}}$$

As we can see labor contribution is here decomposed in two parts. First part, $b_t(\dot{H}/H)$, reflects influence of increase in homogenous labor. Second part, $b_t\sum_{i=0}^{n}(F_{H_{it}}/F_{H_t})\Delta(H_i/H)$, measures contribution of changes in educational structure on economic growth.

Similar result can be derived using production function ***(17)*** or ***(22)***. First, rate of growth of production in those specific cases can be presented as

$$\dot{Q}/Q = \dot{A}/A + a(\dot{K}/K) + b(\dot{H}^*/H^*) \qquad \textbf{\textit{(35)}}$$

Meaning of particular parts of equation is obvious. Applying same procedure on part that measure total contribution of labor, $b(\dot{H}^*/H^*)$, as above we get

$$b(\dot{H}^*/H^*) = b(\dot{H}/H) + b[(\dot{H}^*/H^*) - (\dot{H}/H)]$$

$$b(\dot{H}^*/H^*) = b(\dot{H}/H) + b[\frac{\Delta(H^*/H)}{(H^*/H)}] \qquad \textbf{\textit{(36)}}$$

As in previous case first part measure contribution of homogenous labor, while second part expresses contribution of change in educational structure of labor force to economic growth.

However, contribution of education to economic growth is much larger than sole contribution of change in educational structure. Apart from structural changes it should include contributions of those efforts in education that has been made in order to sustain existing level of education of increasing labor force. This part of educational effect is especially important in those countries that experience high rate of growth of population and labor force. In order to express this effect we will add and subtract contribution of "raw" labor, that is contribution of unskilled part of labor, $b_u(\dot{H}/H)$, in first part of the expression ***(34)***. We get

$$\sum_{i=0}^{n} b_{it}\frac{\dot{H}_i}{H_i} = b_{ut}(\frac{\dot{H}}{H}) + [\frac{(F_{H_t} - F_{H_{0t}})H_t}{Q_t}](\frac{\dot{H}}{H}) + b_t\sum_{i=0}^{n}(\frac{F_{H_{it}}}{F_{H_t}})\Delta(\frac{H_i}{H}) \qquad \textbf{\textit{(37)}}$$

where

$$b_{ut} = \frac{F_{H_{0t}}H_t}{Q_t} \qquad \textbf{\textit{(38)}}$$

stands for elasticity of production with respect to unskilled part of work. In similar manner this effect can be expressed applying same procedure on expression ***(36)*** for specific production function. In that case we get

$$b\left(\frac{\dot{H}^*}{H^*}\right) = b\left(\frac{H_t}{H_t^*}\right)\left(\frac{\dot{H}}{H}\right) + b\left(1 - \frac{H_t}{H_t^*}\right)\left(\frac{\dot{H}}{H}\right) + b\left(\frac{\Delta(H^*/H)}{(H^*/H)}\right) \qquad \textbf{\textit{(39)}}$$

In both expressions, ***(37)*** and ***(39)***, first part of expression presents contribution of "raw" labor. It is important to note that it does not refer to contribution of unskilled workers but to contribution of unskilled part of work of any worker, something that any of us would be able to contribute even without any schooling. Second part, obviously, presents contribution of efforts made to sustain educational level of increasing labor force. Finally, last part, as before, presents influence of change in educational structure of labor force.

So, total contribution of education to economic growth is given as a sum of second and third part of expressions ***(37)*** and ***(39)***. If we now add those two parts of educational contribution we get another interesting and useful decomposition of labor contribution to economic growth. So, in the case of general production function we get

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right)=b_{ut}\left(\frac{\dot{H}}{H}\right)+b_t\sum_{i=1}^{n}\frac{(F_{H_{it}}-F_{H_{0t}})H_{it}}{F_{H_t}H_t}\left(\frac{\dot{H}_i}{H_i}\right)=b_{ut}\left(\frac{\dot{H}}{H}\right)+\sum_{i=1}^{n}\frac{(F_{H_{it}}-F_{H_{0t}})H_{it}}{Q_t}\left(\frac{\dot{H}_i}{H_i}\right) \quad \textbf{\textit{(40)}}$$

Same result can be obtained by adding and subtracting $b_u(\dot{H}/H)$ from expression ***(11)*** for total contribution of labor force to economic growth. If we now mark difference between marginal productivity of *i*-th kind of labor and marginal productivity of unskilled work as $dF_{H_{it}}$ or more formally

$$dF_{H_{it}}=\left(F_{H_{it}}-F_{H_{0t}}\right) \quad \textbf{\textit{(41)}}$$

above expression ***(40)*** becomes

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right)=b_{ut}\left(\frac{\dot{H}}{H}\right)+\sum_{i=1}^{n}\frac{dF_{H_{it}}H_{it}}{Q_t}\left(\frac{\dot{H}_i}{H_i}\right) \quad \textbf{\textit{(42)}}$$

In the case of specific production function ***(17)*** or ***(22)*** using similar procedure we get

$$b\left(\frac{\dot{H}^*}{H^*}\right)=b\left(\frac{H_t}{H_t^*}\right)\left(\frac{\dot{H}}{H}\right)+b\left[\left(\frac{\dot{H}^*}{H^*}\right)-\left(\frac{H_t}{H_t^*}\right)\left(\frac{\dot{H}}{H}\right)\right] \quad \textbf{\textit{(43)}}$$

Note that in last part of expression ***(42)*** we have sum of contributions of every single educational category of workers to economic growth. So, we can measure ED contribution of those with elementary school, those with secondary education, with university degree and so on.

4. Note that expression ***(40)*** makes possible some additional specifications of production function. By substituting ***(40)*** in ***(10)*** we get

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\left(\frac{\dot{H}}{H}\right)+\sum_{i=1}^{n}\frac{dF_{H_{it}}H_{it}}{Q_t}\left(\frac{\dot{H}_i}{H_i}\right) \quad \textbf{\textit{(44)}}$$

4.1. If we now assume constancy of capital shares (a_t=a), "raw" labor share (b_{ut}=b_u), and share of particular kind of ED capital in GDP ($dF_{H_{it}}H_{it}/Q_t=\delta_{it}=\delta_i$) and solve this differential equation (again by integrating and taking antilogarithm) we can get following Cobb-Douglas production function[15]

$$Q_t=A_tH_t^{b_u}K_t^{a}\prod_{i=1}^{n}H_{it}^{\delta_i}=A_tH_t^{b_u}\left[K_t^{\frac{a}{a+\delta}}\prod_{i=1}^{n}H_{it}^{\frac{\delta_i}{a+\delta}}\right]^{a+\delta} \quad \textbf{\textit{(45)}}$$

where $\delta=\sum\delta_i=\sum\frac{dF_{H_{it}}H_{it}}{Q_t}$. This expression is, obviously, analogous to previously derived expression ***(17)*** (or better to its' transformation in expression ***(19)***). While constancy of capital (a_t=a) share in GDP can be regarded as relatively realistic, constancy of different type of ED capital share in GDP (δ_{it}=δ_i) is extremely unrealistic and contradicting to widespread facts that show increase of higher level education share in GDP. Expression ***(45)***, in other word, has same problems

[15] To derive this expression we used manipulation similar to one used previously for expression (17). Meaning of enumerated assumptions is also similar to one used in deriving expression (17).

as previously derived expression *(17)* in analysis and measurement of human / ED capital contribution to economic growth.

4.2. On the other hand, if we assume constancy of capital share ($a_t=a$), "raw" labor share ($b_{ut}=b_u$), total ED capital share in GDP ($\delta_t = \sum \delta_{it} = \sum \frac{dF_{H_{it}} H_{it}}{Q_t} = \delta$), and constancy of relative level of marginal productivity of different kind of ED capital ($\frac{dF_{H_{it}}}{F_{H_{0t}}} = \gamma_{it} = \gamma_i$), and than solve this differential equation (again by integrating and taking antilogarithm) we are getting following production functions[16]

$$Q_t = A_t H_t^{b_u} K_t^a \left[\sum_{i=1}^{n} \gamma_i H_{it} \right]^{\delta} = A_t H_t^{b_u} \left\{ K_t^{\frac{a}{a+\delta}} \left[\sum_{i=1}^{n} \gamma_i H_{it} \right]^{\frac{\delta}{a+\delta}} \right\}^{a+\delta} \tag{46}$$

This expression is, obviously, comparable to previously derived expression *(22)* (or *(27)*). We know from previous consideration that, owing to the rising wage premium that we have witnessed in last three decades, assumption of constancy of relative level of marginal productivity of different kind of ED capital ($\frac{dF_{H_{it}}}{F_{H_{0t}}} = \gamma_{it} = \gamma_i$) cannot be regarded as realistic. So, expression *(46)* has a same problem as expression *(22)* in explaining contribution of human capital to economic growth.

4.3. Expression *(44)* can be also transformed by dividing numerator and denominator of its' second and third part with F_{H0} to get following rate of growth decomposition $\dot{Q}/Q = \dot{A}/A + (a_t + b_{ut})\{\frac{n_{Kt}K_t}{n_{Kt}K_t + n_{0t}H_t}(\dot{K}/K) + \frac{n_{0t}H_t}{n_{K)t}K_t + n_{0t}H_t}(\dot{H}/H)\} + \sum_{i=1}^{n} \delta_{it}(\dot{H}_i/H_i)$

where, as before, $n_{Kt} = \frac{F_{Kt}}{F_{H_0t}}$ and $n_{0t} = \frac{F_{H_0t}}{F_{H_0t}} = 1$. Assuming now constancy of all relevant parameters, that is $n_{Kt} = n_K$, $(a_t + b_{ut}) = (a + b_u)$, and $\delta_{it}=\delta_i$, than solving this differential equation gives

$$Q_t = A_t (n_K K + H_t)^{(a+b_u)} \prod_{i=1}^{n} H_{it}^{\delta_i} \tag{47}$$

which in the case of only two kind of labor, skilled (H_0) and unskilled (H_1), becomes

$$Q_t = A_t (n_K K + H_{0t} + H_{1t})^{(a+b_u)} H_{1t}^{\delta_1} \tag{48}$$

This expression resembles well-known Griliches (1969) function, and, for that reason, might have power to explain rising wage premium ratio. Even more power to explain rising wage premium ratio would have nested CES function that may be derived using this framework.

4.4. If we, on the other hand, divide all parts of expression *(44)* with F_{H0} and assume constancy of all relevant parameters, that is $\gamma_{it} = \gamma_i$, $n_{Kt} = \frac{F_{Kt}}{F_{H_0t}} = n_K$, $\delta_t = \delta$, $b_{ut} = b_u$, and $a_t = a$, than by solving this differential equation, knowing that $a + b_u + \delta = 1$, we get

[16] In deriving this expression we used manipulation similar to one used for expression (22). Meaning of particular assumptions is also similar to one used in deriving expression (22).

$$Q_t = A_t(a + b_u + \delta)\left[n_K K_t + H_t + \sum_{i=1}^{n}\gamma_i H_{it}\right] = A_t\left[n_K K_t + H_t + \sum_{i=1}^{n}\gamma_i H_{it}\right] = A_t \overline{K}_t \qquad \textbf{\textit{(49)}}$$

This, again, can be regarded as a specific form of linear production function or as a specific form of "AK" function.

5. It is even more interesting to express and measure contribution of different level of schooling, that is contribution of productive power reached at each particular level of schooling to economic growth[17]. To do it note, first, that

$$dF_{H_{it}} = \left(F_{H_{it}} - F_{H_{0t}}\right) = mdF_{H_{1t}} + mdF_{H_{2t}} + \ldots + mdF_{H_{vt}} + \ldots mdF_{H_{it}} = \sum_{v=1}^{i} mdF_{H_{vt}} \qquad \textbf{\textit{(50)}}$$

where

$$mdF_{H_{it}} = dF_{H_{it}} - dF_{H_{(i-1)t}}$$

$$\textbf{\textit{(51)}}$$

presents difference in marginal productivity (wages) of two successive levels (years) of education. Having that in mind expression ***(42)*** can be transformed in the following form[18]

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right) = b_{ut}\left(\frac{\dot{H}}{H}\right) + \sum_{i=1}^{n}\frac{mdF_{H_{it}}R_{it}}{Q_t}\left(\frac{\dot{R}_i}{R_i}\right) \qquad \textbf{\textit{(52)}}$$

where

$$R_{it} = \sum_{v=i}^{n} H_{vt} \qquad \textbf{\textit{(53)}}$$

presents number of workers who have *i*-th and higher level of education. Obviously when we multiply, like in this expression, number of all workers that have that particular level and higher levels of education with difference between marginal productivity of that and previous level of education and than multiply that result with relative increase in R_{it} what we get is contribution of education reached at *i*–th level of schooling to economic growth. So, each part of last term, $(mdF_{H_{it}} R_{it} / Q_t)(\dot{R}_i / R_i)$, measure contribution of each level of educational system to economic growth.

6. Note at the end that expression ***(43)*** also makes possible some additional specifications of production function. By substituting ***(52)*** in ***(10)*** we get

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[17] For details see for example Psacharopoulos, G. (1972).

[18] Part that measure contribution of education can be transformed in the following way

$$\sum_{i=1}^{n}\left(dF_{H_{it}} H_{it} / Q_t\right)\left(\dot{H}_i / H_i\right) = (1/Q_t)\left[dF_{H_{1t}} H_{1t}(\dot{H}_1 / H_1) + dF_{H_{2t}} H_{2t}(\dot{H}_2 / H_2) + \ldots + dF_{H_{nt}} H_{nt}(\dot{H}_n / H_n)\right] =$$

$$(1/Q_t)\left\{\begin{matrix} mdF_{H_{1t}} H_{1t}(\dot{H}_1 / H_1) + \left[mdF_{H_{1t}} H_{2t}(\dot{H}_2 / H_2) + mdF_{H_{2t}} H_{2t}(\dot{H}_2 / H_2)\right] + \ldots + \\ \left[mdF_{H_{1t}} H_{nt}(\dot{H}_n / H_n) + mdF_{H_{2t}} H_{nt}(\dot{H}_n / H_n) + \ldots + mdF_{H_{nt}} H_{nt}(\dot{H}_n / H_n)\right] \end{matrix}\right\} =$$

$$(1/Q_t)\left\{\begin{matrix} mdF_{H_{1t}}\left[H_{1t}(\dot{H}_1 / H_1) + H_{2t}(\dot{H}_2 / H_2) + \ldots + H_{nt}(\dot{H}_n / H_n)\right] + \\ mdF_{H_{2t}}\left[H_{2t}(\dot{H}_2 / H_2) + \ldots + H_{nt}(\dot{H}_n / H_n)\right] + \ldots + mdF_{H_{nt}} H_{nt}(\dot{H}_n / H_n) \end{matrix}\right\} =$$

$$(1/Q_t)\left[mdF_{H_{1t}} R_{1t}(\dot{R}_1 / R_1) + mdF_{H_{2t}} R_{2t}(\dot{R}_2 / R_2) + \ldots + mdF_{H_{nt}} R_{nt}(\dot{R}_n / R_n)\right] =$$

$$\sum_{i=1}^{n}\frac{mdF_{H_{it}} R_{it}}{Q_t}\left(\frac{\dot{R}_i}{R_i}\right)$$

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\left(\frac{\dot{H}}{H}\right)+\sum_{i=1}^{n}\frac{mdF_{H_{it}}R_{it}}{Q_t}\left(\frac{\dot{R}_i}{R_i}\right) \qquad \textbf{\textit{(54)}}$$

6.1. If we now again assume constancy of capital shares (a_t=a), "raw" labor share (b_{ut}=b_u), and share of particular kind of ED capital in GDP ($mdF_{H_{it}}R_{it}/Q_t=\beta_{it}=\beta_i$) and solve this differential equation (again by integrating and taking antilogarithm) we can get following Cobb-Douglas production function[19]

$$Q_t=A_tH_t^{b_u}K_t^{a}\prod_{i=1}^{n}R_{it}^{\beta_i}=A_tH_t^{b_u}\left[K_t^{\frac{a}{a+\beta}}\prod_{i=1}^{n}R_{it}^{\frac{\beta_i}{a+\beta}}\right]^{a+\beta} \qquad \textbf{\textit{(55)}}$$

where $\beta=\sum\beta_i=\sum\frac{mdF_{H_{it}}R_{it}}{Q_t}$. This expression is, obviously, comparable to previously derived expression ***(17)*** and expression ***(45)***. Again, while constancy of capital share in GDP (a_t=a) can be regarded as relatively realistic and acceptable, constancy of different type of ED capital share in GDP (β_{it}=β_i) is extremely unrealistic and contradicting to empirical facts. Expression ***(55)***, in other word, has same problems as previously derived expressions ***(17)*** and ***(45)*** in analysis and measurement of human / ED capital contribution to economic growth.

6.2. On the other hand, however, if we assume constancy of capital share (a_t=a), "raw" labor share (b_{ut}=b_u), total ED capital share in GDP $(\beta_t=\sum\beta_{it}=\sum\frac{mdF_{H_{it}}R_{it}}{Q_t}=\beta)$, and constancy of relative level of marginal productivity of different kind of ED capital ($\frac{mdF_{H_{it}}}{F_{H_{0t}}}=\alpha_{it}=\alpha_i$), and than solve this differential equation (again by integrating and taking antilogarithm) we are getting following production functions[20]

$$Q_t=A_tH_t^{b_u}K_t^{a}\left[\sum_{i=1}^{n}\alpha_iR_{it}\right]^{\beta}=A_tH_t^{b_u}\left\{K_t^{\frac{a}{a+\beta}}\left[\sum_{i=1}^{n}\alpha_iR_{it}\right]^{\frac{\beta}{a+\beta}}\right\}^{a+\beta} \qquad \textbf{\textit{(56)}}$$

This expression is, obviously, analogous to previously derived expressions ***(22)*** (or ***(27)***) and ***(46)***. We know from previous consideration that above assumptions cannot be regarded as realistic. So, expression ***(56)*** seems not to be acceptable for analysis and measurement of human capital contribution to economic growth.

6.3. In the similar manner as in previous case it is possible to derive following Griliches wise production function

$$Q_t=A_t(n_KK+H_t)^{(a+b_u)}\prod_{i=1}^{n}H_{it}^{\beta_i} \qquad \textbf{\textit{(57)}}$$

which in the case of only two kind of labor, skilled (H_0) and unskilled (H_1), becomes

$$Q_t=A_t(n_KK+H_{0t}+H_{1t})^{(a+b_u)}H_{1t}^{\beta_1} \qquad \textbf{\textit{(58)}}$$

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[19] Like before, to derive of this expression we used manipulation similar to one used previously for expression (17). Meaning of enumerated assumptions is also similar to one used in deriving expression (17) and (45).

[20] Again, in deriving this expression we used manipulation similar to one used for expression (22). Meaning of particular assumptions is also similar to one used in deriving expression (22) and (46).

This expression is able to capture capital complementarity effect and in that way to explain rising wage premium ratio. In the similar manner it is possible to derive adequate nested CES function that has even better chance to explain rising wage premium ratio.

6.4. Finally, we can, following similar manipulation and assumptions like in previous section, derive linear and / or “AK” production function of the form

$$Q_t = A_t\left[n_K K_t + H_t + \sum_{i=1}^{n}\alpha_i H_{it}\right] = A_t \overline{K}_t \quad \textbf{(59)}$$

3. Models with Heterogeneous Capital of Education (ED)

1. In order to develop model with capital of education we start with general production function of the form

$$Q_t = F(K_t, H_t, E_{1t}, E_{2t} ... E_{it} ... E_{nt}, t) \quad \textbf{(60)}$$

Here, as before, K_t stands for capital and t present time. However, H_t now presents unskilled part of work measured in hours of works of all workers. E_{it} is new symbol and it represents quantity of educational capital “owned” by those with i-th level (or kind) of education or i-th years of schooling. Formally

$$E_{it} = H_{it} l_i \quad \textbf{(61)}$$

where l_i presents quantity of educational capital per worker of particular level / kind of education. In order to simplify analysis, we will assume that this value is constant over time. However, average capital of education per capita is not constant and can be presented as

$$l_t = \sum_{i=1}^{n} l_i \left(H_{it} / H_t\right) \quad \textbf{(62)}$$

Finally, total quantity of educational capital is given by

$$E_t = \sum_{i=1}^{n} E_{it} = \sum_{i=1}^{n} H_{it} l_i = H_t \sum_{i=1}^{n} l_i (H_{it} / H_t) = H_t l_t \quad \textbf{(63)}$$

As far as *measurement of human and educational capital* is regarded two different approaches have been proposed so far in economic literature. First one, which will be followed in this article, is *cost-based approach*[21]. Basically capital of education is here measured with cost of reaching particular level / kind of education. Not only direct cost of schooling (books, transportation, tuition fees and other), but also all opportunity costs are supposed to be taken into account. In fact, opportunity costs in the form of students foregone earning are most important part of those costs, and they can make from 70% to 80% of all costs of reaching particular level of education. Note also that not only private (individual and household), but also all social costs are supposed to be captured for this kind of analysis.

Second is *income-based approach*[22]. Simply speaking, capital of education and human capital are here calculated as present value of stream of benefits (increased earning) generated by investment in particular kind of education or human capital in general. What we need for this kind of measurement is appropriate discount rate. It should be equal to the *required rate of return* for investment in particular kind of education. Required rate of return, on the other hand, should be calculated using appropriate risk premium for investment in particular kind of education.

[21] This approach has been most prominently advocated in works of Shultz, T. W. (1960, 1961a, 1961b, 1962), Kendrick, J. W. (1976), and Eisner, R. (1985, 1988). For a good survey see Griliches, Z. (1996).

[22] Especially important here are works of Jorgenson, D. W. and Fraumeni, B. M. (1989, 1992a, 1992b). See also Fraumeni, B. M. (2000) and Fraumeni, M. B., Reinsdorf, M. B., Robinson, B. B., Williams, M. P. (2004a, 2004b).

Unfortunately this rate is not easy to establish so that different authors use in their calculations arbitrarily taken discount rates. Note that in equilibrium two measures of ED capital, cost-based and income-based (with required rate of return), should be equal, and that required rate of return should be equal to internal rate of return. Since effects of investment in education are long lasting (40 years and more) and since technological progress constantly changes demand for different kind of education, equilibrium is almost impossible to be reached in activity like education. Consequently, we can always expect to have discrepancy between required rate of return and internal rate of return in this kind of investment. On the other hand, owing to the widespread externalities, we can also expect to have constant discrepancy between social rates of return and private rates of returns in education. Having all this in mind, it is obviously much better to rely on cost-based approach in measuring capital of education. Fitting of production function loaded with cost-based capital of education is likely to give, among other things, an estimate of social rate of return on investment in education[23]. This in turn should help us to establish right measure of ED capital contribution to economic growth.

Unfortunately, owing to the lack of data necessary to calculate capital of education, most of the authors have used so far *average number of years of schooling* as a *proxy for capital of education per capita*. Indeed, if we assume that costs of reaching additional year of schooling are constant and equal for every level (year) of schooling, than it can be shown that this proxy is quite appropriate[24].

2. By differentiating expression ***(60)*** and dividing with $\boldsymbol{Q_t}$ we are getting rate of growth of production decomposed in the following way

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+\left(\frac{F_{H_0}H_t}{Q_t}\right)\frac{\dot{H}}{H}+\sum_{i=1}^{n}f_{it}\left(\frac{\dot{E}_i}{E_i}\right)=$$
$$=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}f_{it}\left(\frac{\dot{E}_i}{E_i}\right) \qquad \textbf{\textit{(64)}}$$

It is obvious that first element, $\dot{A}/A$, presents contribution of global factor productivity to growth rate of GDP, second element, $a_t(\dot{K}/K)$, measures contribution of capital accumulation, third part, $b_{ut}(\dot{H}/H)$, measure contribution of "raw" labor, while last element, $\sum_{i=1}^{n}f_{it}\left(\dot{E}_i/E_i\right)$, express contribution of all types of educational capital to the rate of growth.

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[23] Social rate of return estimated in this way is supposed to capture all kinds of externalities, not only those covered by social internal rate of return used in ordinary Cost-Benefit analysis. It, therefore, should be much larger than social internal rate of return provided by Cost-Benefit analysis of investment in education. See Sianesi, B., Van Reenen, J. (2000, 2002).

[24] If we assume that per capita *cost of reaching additional*, *i*-th *year of schooling* (g_{it}) are constant and equal for all level of schooling, that is

$$g_{it}=l_{it}-l_{(i-1)t}=l_i-l_{(i-1)}=g_i=\bar{g}$$

than *total per capita cost of reaching i-th year of education* can be approximated as $l_i=i\bar{g}$

Substituting now this in expression (62) for capital of education per capita we can show that

$$l_t=\sum_{i=1}^{n}l_i(H_{it}/H_t)=\sum_{i=1}^{n}i\bar{g}(H_{it}/H_t)=\bar{g}\sum_{i=1}^{n}i(H_{it}/H_t)=\bar{g}y_t$$

where, obviously, $y_t=\sum i(H_{it}/H_t)$ presents average year of schooling. Since $\bar{g}$ is constant by assumption, than obviously rate of growth (or index of growth) of capital of education per capita will be equal to rate of growth of average years of schooling.

As before, coefficient $a_t = F_{Kt} K_t / Q_t$ presents elasticity of production with respect to capital, b_{ut} is elasticity of production with respect to "raw" labor, while $f_{it} = (F_{Eit} E_{it} / Q_t)$ presents elasticity of production with respect to *i*-th kind of educational capital. As usual, $F_{Kt} = \partial Q/\partial K$ stands for marginal productivity of capital while $F_{Eit} = \partial Q/\partial E_i$ stands for marginal productivity of *i*-th type of educational capital. Having in mind expression ***(61)*** and previous considerations we can state that

$$F_{E_{it}} = \frac{\partial Q}{\partial E_i} = \frac{\left(F_{H_{it}} - F_{H_{0t}}\right)}{l_i} = \frac{dF_{H_{it}}}{l_i} \qquad \textbf{\textit{(65)}}$$

and this is even intuitively understandable. It is important to notice that, although dF_{Hit} should be larger for higher levels of education (for larger *i*), same does not apply for F_{Eit} because it measure marginal productivity (rate of return) of money invested in particular level of education and not marginal productivity of hours of work of that level of education: it can easily happen that money invested in elementary literacy be more productive than money invested in university education[25].

Now based on ***(61)*** and ***(65)*** it can be shown that

$$f_{it} = \frac{F_{E_{it}} E_{it}}{Q_t} = \frac{\left[\left(\frac{dF_{H_{it}}}{l_i}\right) H_{it} l_i\right]}{Q_t} = \frac{dF_{H_{it}} H_{it}}{Q_t} \qquad \textbf{\textit{(66)}}$$

and

$$\frac{\dot{E}_i}{E_i} = \frac{\dot{H}_i}{H_i} + \frac{\dot{l}_i}{l_i} = \frac{\dot{H}_i}{H_i} \qquad \textbf{\textit{(67)}}$$

Combining those results it is obvious that

$$f_{it} \frac{\dot{E}_i}{E_i} = \left(\frac{dF_{H_{it}} H_{it}}{Q_t}\right) \frac{\dot{H}_i}{H_i}$$

and

$$\sum_{i=1}^{n} f_{it} \frac{\dot{E}_i}{E_i} = \sum_{i=1}^{n} \left(\frac{dF_{H_{it}} H_{it}}{Q_t}\right)\left(\frac{\dot{H}_i}{H_i}\right) \qquad \textbf{\textit{(68)}}$$

Now adding contribution of "raw" labor to both side of ***(68)*** we arrive at

$$b_{ut} \frac{\dot{H}}{H} + \sum_{i=1}^{n} f_{it} \frac{\dot{E}_i}{E_i} = b_{ut} \frac{\dot{H}}{H} + \sum_{i=1}^{n} \left(\frac{dF_{H_{it}} H_{it}}{Q_t}\right)\left(\frac{\dot{H}_i}{H_i}\right) \qquad \textbf{\textit{(69)}}$$

Comparing expression ***(69)*** with expression ***(42)*** we can conclude that capital of education approach decomposes contribution of labor input in exactly the same way as previous approach that uses hours of work with different levels of education as inputs in production function.

3. The fact that both approaches decompose contribution of education to economic growth in exactly the same way and the fact that educational capital approach assume much more unnecessary computations explain why approach with capital of education has not been used so often in sources of growth analysis. It is not to say that there is a shortage of researches about

[25] Having in mind above considerations, especially relations (63) and (65), we can conclude that decomposition of rate of growth given in formula (64) could have been derived without reference to the type of general production function given in (60). By multiplying and dividing second part in expression (40) with l_i and substituting $(\dot{E}_i / E_i)$ for $(\dot{H}_i / H_i)$ in it (in accordance with expression (67)), we are able to transform contribution of education from it's original exposition (second part of (40)) to one given in last part of expression (64).

efficiency of investment in education. However, most of those researches have been in the microeconomics and in the field of cost benefit analysis of educational investments, and rarely in the field of sources of growth analysis[26]. We will now show that this approach makes possible specific decomposition of contribution of education, one that is not possible with Denison's like approach, and that for that reason this kind of sources of growth analysis can be very useful indeed. If we add and subtract value of $f_t(\dot{E}/E)$ to the second part of expression ***(69)*** and transform it[27], we get

$$b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n} f_{it}\frac{\dot{E}_i}{E_i}=b_{ut}\frac{\dot{H}}{H}+f_t\frac{\dot{H}}{H}+f_t\frac{\dot{l}}{l}+f_t\sum_{i=1}^{n}\left(\frac{F_{E_{it}}}{F_{E_t}}\right)\Delta\left(\frac{E_i}{E}\right) \qquad (70)$$

where

$$f_t=\sum_{i=1}^{n}\frac{F_{E_{it}}E_{it}}{Q_t}=\frac{F_{E_t}E_t}{Q_t} \qquad (71)$$

presents elasticity of production with respect to aggregate capital of education. As before first part measure contribution of "raw" labor. Keeping in mind ***(71)*** and ***(66)***, second part can be transformed to

$$f_t\frac{\dot{H}}{H}=\sum_{i=1}^{n}\left(\frac{F_{E_{it}}E_{it}}{Q_t}\right)\left(\frac{\dot{H}}{H}\right)=\sum_{i=1}^{n}\left[\frac{\left(F_{H_{it}}-F_{H_{0t}}\right)H_{it}}{Q_t}\right]\left(\frac{\dot{H}}{H}\right) \qquad (72)$$

From ***(37)*** we already know that it presents contribution of efforts made to sustain educational level of increasing labor force. But, what is than meaning of last two parts of expression ***(70)***?

Comparing expression ***(70)*** and ***(37)*** we can establish following important relation

$$b_t\sum_{i=1}^{n}\left(\frac{F_{H_{it}}}{F_{H_{jt}}}\right)\Delta\left(\frac{H_i}{H}\right)=f_t\frac{\dot{l}}{l}+f_t\sum_{i=1}^{n}\left(\frac{F_{E_{it}}}{F_{E_t}}\right)\Delta\left(\frac{E_i}{E}\right) \qquad (73)$$

We now see that contribution of change in educational structure of labor force from previous chapter, using ED capital approach can be broken up into two parts. This is something that is not possible to convey using Denison's approach in sources of growth analysis and it

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[26] Earliest works that use concept of ED capital for this purposes are these of Schultz T. (1960, 1962), Kendrick, J. W. (1979, 1980, 1981), Griliches, Z. V. (1963b, 1964). See also Bowman, M. J. (1964).

[27] It is obvious that

$$\sum_{i=1}^{n} f_{it}(\dot{E}_i/E_i)=f_t(\dot{E}/E)+f_t\sum_{i=1}^{n}\frac{F_{E_{it}}E_{it}}{F_{E_t}E_t}\left(\frac{\dot{E}_i}{E_i}-\frac{\dot{E}}{E}\right)\approx f_t(\dot{E}/E)+f_t\sum_{i=1}^{n}\frac{F_{E_{it}}E_{it}}{F_{E_t}E_t}\left(\frac{\Delta(E_i/E)}{(E_i/E)}\right)=$$

$$f_t(\dot{E}/E)+f_t\sum_{i=1}^{n}\left(\frac{F_{E_{it}}}{F_{E_t}}\right)\Delta\left(\frac{E_i}{E}\right)$$

Now, since $E = H\,l$ it follow that $f_t(\dot{E}/E)=f_t(\dot{H}/H)+f_t(\dot{l}/l)$ so that

$$\sum_{i=1}^{n} f_{it}(\dot{E}_i/E_i)=f_t(\dot{H}/H)+f_t(\dot{l}/l)+f_t\sum_{i=1}^{n}\frac{F_{E_{it}}}{F_{E_t}}\Delta\left(\frac{E_i}{E}\right)$$

Finally

$$b_{ut}(\dot{H}/H)+\sum_{i=1}^{n} f_{it}(\dot{E}_i/E_i)=b_{ut}(\dot{H}/H)+f_t(\dot{H}/H)+f_t(\dot{l}/l)+f_t\sum_{i=1}^{n}\frac{F_{E_{it}}}{F_{E_t}}\Delta\left(\frac{E_i}{E}\right)$$

presents important advantage of this methodology. Meaning of first part is intuitively clear: it captures contribution of change in educational structure of labor force; more precisely it presents contribution of increase of capital of education per capita. Greater increase of capital of education per capita implies stronger improvement of educational structure of labor force, and it implies greater rate of growth. It is much clearer if we transform this element further following ***(62)***[28]

$$f_t \frac{\dot{l}}{l} = f_t \sum_{i=1}^{n} \left(\frac{l_i}{l} \right) \Delta \left(\frac{H_i}{H} \right) \qquad (74)$$

Obviously, increase in labor share, $\Delta(H_i/H)>0$, of those who "own" greater per capita capital of education than average, $(l_i/l)>1$, have same meaning as improvement of labor force structure. But this improvement and corresponding increase in educational capital per capita can be done in more or less effective way, and this is something that is supposed to be captured with second part of expression ***(73)***. As we see, this part measures improvement in structure of educational capital. Increase in relative size, $\Delta(E_i/E)>0$, of those categories of educational capital that have above average relative productivity, $(F_{Eit}/F_{Et})>1$, will have positive influence on economic growth, and vice versa. As we suggested earlier it is quite possible that increase of those with basic literacy (followed with decrease of illiterate) be more efficient way of bettering educational structure than increase of university graduate (followed with decrease of those with secondary education).

4. It is now possible to give one additional analysis of contribution of education to economic growth based on a concept of educational capital. This time concept of educational capital is defined in little bit different way. Notice, first, that per capita costs of reaching certain additional level (year) of education, ***i***, from previous one, *i-1*, are given by

$$g_i = l_i - l_{(i-1)} \qquad (75)$$

Since by definition $(\Delta l_i/l_i) = 0$ for every *i*, it follow that

$$\frac{\dot{g}_i}{g_i} = 0 \qquad (76)$$

Capital of education reached at certain level of education (year of schooling) can obviously now be defined as

$$G_{it} = g_i R_{it} \qquad (77)$$

where R_{it}, as before (see expression ***(53)***), presents number of workers who have *i*-th and higher level of education. In other words, it presents number of all workers who attended this particular level of education no matter whether they continue their education latter or not. From ***(76)*** and ***(77)*** it follow that

$$\frac{\dot{G}_i}{G_i} = \frac{\dot{R}_i}{R_i} \qquad (78)$$

If we now take decomposition of total labor hours contribution given in expression ***(52)***

[28] Following $l = \sum l_i (H_{it} / H_t)$ from (62) we can write

$$f_t(\dot{l}/l) \approx f_t \sum_{i=1}^{n} \frac{l_i H_{it}}{l H_t} \left[\left(\frac{\dot{l}_i}{l_i} \right) + \left(\frac{\Delta(H_{it}/H_t)}{(H_{it}/H_t)} \right) \right]$$

Now since by definition $(\dot{l}_i / l_i) = 0$ it follows that $f_t(\dot{l}/l) \approx f \sum_{i=1}^{n} \left(\frac{l_i}{l} \right) \Delta \left(\frac{H_{it}}{H_t} \right)$

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right) = b_{ut}\left(\frac{\dot{H}}{H}\right) + \sum_{i=1}^{n}\left(\frac{mdF_{H_{it}} R_{it}}{Q_t}\right)\left(\frac{\dot{R}_i}{R_i}\right)$$

and than multiply and divide its second part with g_i, than, having in mind ***(78),*** we get

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right) = b_{ut}\left(\frac{\dot{H}}{H}\right) + \sum_{i=1}^{n}\left[\frac{\left(mdF_{H_{it}} / g_i\right) R_{it} g_i}{Q_t}\right]\left(\frac{\dot{G}_i}{G_i}\right) \qquad \textbf{\textit{(79)}}$$

Now, it is obvious that new term within summation

$$\frac{mdF_{H_{it}}}{g_i} = F_{G_{it}} \qquad \textbf{\textit{(80)}}$$

presents marginal productivity of newly defined capital of education. It is marginal productivity of capital invested in particular level / year of schooling. Substituting in previous expression and having in mind ***(77)***, we arrive at

$$\sum_{i=1}^{n} b_{it}\left(\frac{\dot{H}_i}{H_i}\right) = b_{ut}\frac{\dot{H}}{H} + \sum_{i=1}^{n}\left(\frac{F_{G_{it}} G_{it}}{Q_t}\right)\left(\frac{\dot{G}_i}{G_i}\right) \qquad \textbf{\textit{(81)}}$$

Second part of this formula now presents total contribution of education to economic growth. On the other hand, each part of this summation, $(F_{G_i} G_i / Q)(\dot{G}_i / G_i)$, presents contribution of particular level of education to economic growth.

Substituting now value of $\sum_{i=1}^{n} b_{it}(\dot{H}_i / H_i)$ from ***(81)*** in ***(10)*** we get following decomposition of rate of growth of production

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + b_{ut}\frac{\dot{H}}{H} + \sum_{i=1}^{n}\left(\frac{F_{G_{it}} G_{it}}{Q_t}\right)\left(\frac{\dot{G}_i}{G_i}\right) = \frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + b_{ut}\frac{\dot{H}}{H} + \sum_{i=1}^{n} q_{it}\left(\frac{\dot{G}_i}{G_i}\right) =$$

$$\frac{\dot{A}}{A} + a_t\frac{\dot{K}}{K} + b_{ut}\frac{\dot{H}}{H} + q_t\sum_{i=1}^{n}\left(\frac{F_{G_{it}} G_{it}}{F_{G_t} G_t}\right)\left(\frac{\dot{G}_i}{G_i}\right) \qquad \textbf{\textit{(82)}}$$

where $q_{it} = F_{G_{it}} G_{it} / Q_t$ presents elasticity of production with respect to *i*-th level of educational capital, while $q_t = \sum F_{G_{it}} G_{it} / Q_t = F_{G_t} G_t / Q_t$ presents elasticity of production with respect to aggregate capital of education. Meaning of each part of this expression is pretty obvious: first part measure contribution of technological progress to economic growth, second part express contribution of capital accumulation, third part is influence of "raw" labor, and, finally, last part measure contribution of capital of education to economic growth. Notice, however, that same result could have been established if we had started with general production function similar to one given in expression ***(60)*** but with capital of education defined like in expression ***(77)*** instead of E_{it}.

Notice also that, using similar procedure as before for E_{it}, we can decompose contribution of education further and get some other interesting results. Especially important might be following relation

$$b_t\sum_{i=1}^{n}\left(\frac{F_{H_{it}}}{F_{H_t}}\right)\Delta\left(\frac{H_i}{H}\right) = f_t\frac{\dot{l}}{l} + f_t\sum_{i=1}^{n}\left(\frac{F_{G_{it}}}{F_{G_t}}\right)\Delta\left(\frac{G_i}{G}\right) \qquad \textbf{\textit{(83)}}$$

It is very similar to relation ***(73)*** established earlier. As a matter of fact it is derived in very similar way[29] as expression ***(73)***. More importantly, it convey same idea in even more obvious way: contribution of improvement in educational structure of a labor force can be split into two parts; first part, as before, measure contribution of increase of per capita capital of education, while second part measure contribution of structural improvement of ED capital. Meaning of this second part is now much clearer than before. It is now much clearer that increase of those with basic literacy can be better way to improve educational structure than increase of university graduates, provided, of course, that marginal productivity of investment in literacy is higher than marginal productivity of investment in university education.

5. Let us now go back to expression ***(64)***

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}f_{it}\left(\frac{\dot{E}_i}{E_i}\right)=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\frac{\dot{H}}{H}+f_t\sum_{i=1}^{n}\left(\frac{F_{Eit}E_{it}}{\sum F_{Eit}E_{it}}\right)\left(\frac{\dot{E}_i}{E_i}\right)$$

and see what can happen if we take different assumption about behavior of its parameters.

5.1. If we, first, assume that elasticity of substitution between any two kinds of factors of production is equal to one and independent of quantity of other factors, it will allow us to simplify initial production function. Here more precisely it means that, first, elasticity of substitution between capital, K_t, and any kind capital of education, E_{it} for any i, is equal to one and independent of quantities of other inputs. In other words, any change of marginal rate of substitution between

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[29] By adding and subtracting $q_t(\dot{G}/G)=\sum q_{it}(\dot{G}/G)$, expression (81) can be transformed in the following way

$$\sum_{i=0}^{n}b_{it}\left(\frac{\dot{H}_i}{H_i}\right)=b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}\left(\frac{F_{G_{it}}G_{it}}{Q_t}\right)\left(\frac{\dot{G}_i}{G_i}\right)\approx b_{ut}\frac{\dot{H}}{H}+q_t\frac{\dot{G}}{G}+q_t\sum_{i=1}^{n}\frac{F_{G_{it}}G_{it}}{F_{G_t}G_t}\left[\frac{\Delta(G_i/G)}{(G_i/G)}\right]=$$

$$b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}q_{it}\frac{\dot{G}}{G}+q_t\sum_{i=1}^{n}\frac{F_{G_{it}}}{F_{G_t}}\Delta\left(\frac{G_i}{G}\right)$$

Although $E_{it}\neq G_{it}$, we know that

$$G_t=E_t=\sum_{i=1}^{n}E_{it}=\sum_{i=1}^{n}H_{it}l_{it}=H_tl_t$$

so that

$$\frac{\dot{G}}{G}=\frac{\dot{H}}{H}+\frac{\dot{l}}{l}$$

Notice also that, relying on transformations similar to one given in footnote (17) (this time going in reverse direction), it can be shown that

$$q_t=\sum_{i=1}^{n}q_{it}=\sum_{i=1}^{n}\frac{F_{Git}G_{it}}{Q_t}=\sum_{i=1}^{n}\frac{(mdF_{H_{it}}/g_i)(R_{it}g_i)}{Q_t}=\sum_{i=1}^{n}\frac{mdF_{H_{it}}R_{it}}{Q_t}=\sum_{i=1}^{n}\frac{dF_{H_{it}}H_{it}}{Q_t}=$$

$$\sum_{i=1}^{n}\frac{(dF_{H_{it}}/l_i)(H_{it}l_i)}{Q_t}=\sum_{i=1}^{n}\frac{F_{E_{it}}E_{it}}{Q_t}=\sum_{i=1}^{n}f_{it}=f_t$$

Now by substituting above relations in first expression it can be shown that

$$\sum_{i=1}^{n}b_{it}\frac{\dot{H}}{H}=b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}\frac{dF_{H_{it}}H_{it}}{Q_t}\left(\frac{\dot{H}}{H}\right)+\sum_{i=1}^{n}f_{it}\left(\frac{\dot{l}}{l}\right)+q_t\sum_{i=1}^{n}\frac{F_{G_{it}}}{F_{G_t}}\Delta\left(\frac{G_i}{G}\right)=$$

$$b_{ut}\frac{\dot{H}}{H}+f_t\left(\frac{\dot{H}}{H}\right)+f_t\left(\frac{\dot{l}}{l}\right)+q_t\sum_{i=1}^{n}\frac{F_{G_{it}}}{F_{G_t}}\Delta\left(\frac{G_i}{G}\right)$$

Again, we have that first part of right hand side measure contribution of "raw" labor while second part (see expression (72) measure contribution of those efforts in education made to sustain existing educational level of increasing labor force. Comparing again this relation with relation (37) it is easy to establish relation (83).

capital and particular kind of educational capital, F_K / F_{Ei} for any *i,* is followed with change of specific ratio of two kinds of capital, K / E_i, of a same proportion. Second, elasticity of substitution between different kinds of educational capital is equal to one and independent of quantity of other kinds of capital: any change in marginal rate of substitution between different kinds of capital of education, F_{Ei} / F_{Ej} for any *i* and *j,* is followed with proportional change in ratio of those kinds of capital of education, E_i / E_j for any *i* and *j*. Third, elasticity of substitution between "raw" labor and any kind of capital (educational or physical) is equal to one independently of quantity of other factors. Consequence of those three assumptions is constancy of factors elasticity of production, that is constancy of elasticity of production with respect to capital, a_t, = *a***,** constancy of elasticity of production with respect to any kind of ED capital, $f_{it} = f_i$**,** and constancy of elasticity of production with respect to "raw" labor, $b_{ut} = b_u$**.** This further implies constancy of derived values of f_t (= f) and r_i, where $r_i = F_{Ei}\, E_i / {}_1\sum^n F_{Ei}\, E_i$**.**

Substituting those new values in expression ***(64)*** we get

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + a\frac{\dot{K}}{K} + b_u\frac{\dot{H}}{H} + \sum_{i=1}^{n} f_i\left(\frac{\dot{E}_i}{E_i}\right) \qquad \textbf{\textit{(84)}}$$

If we now solve this as differential equation (by integrating and taking antilogarithm) we obtain

$$Q_t = A_t H_t^{b_u} K_t^{a} \prod_{i=1}^{n} E_{it}^{f_i} = A_t H_t^{b_u} K_t^{a}\left(\prod_{i=1}^{n} E_{it}^{r_i}\right)^{f} = A_t H_t^{b_u} K_t^{a} E^{*f}_{t} \qquad \textbf{\textit{(85)}}$$

where

$$E_t^{*} = \prod_{i=1}^{n} E_{it}^{r_i} \qquad \textbf{\textit{(86)}}$$

presents capital of education measured in same efficiency units. Capital of education is here aggregated using geometric index. All other factors of production are also aggregated using geometric index, or, to put it in other words, like in CD production function. It is multifactor CD production function. Following transformations of ***(85)*** are also interesting

$$Q_t = A_t H_t^{b_u}\left(K_t^{\frac{a}{(a+f)}} \prod_{i=1}^{n} E_{it}^{\frac{f_i}{(a+f)}}\right)^{(a+f)} = A_t H_t^{b_u}\left(K_t^{\frac{a}{(a+f)}} E_t^{*\frac{f}{(a+f)}}\right)^{(a+f)} = A_t H_t^{b_u} C^{(a+f)} \qquad \textbf{\textit{(87)}}$$

where

$$C_t = K_t^{\frac{a}{(a+f)}} E_t^{*\frac{f}{(a+f)}} \qquad \textbf{\textit{(88)}}$$

presents total capital, tangible (K_t) and intangible (E_t), combined using geometric index. This capital is here powered with sum of ordinary capital and ED capital shares. Therefore in this case we are again totally in the world of CD production function. Notice that expressions ***(85)*** and ***(87)*** resemble very much to one given in expression ***(2)*** and used by Mankiw et al (1992). However it is not the same: total or aggregate ED capital is here obtained using geometric index; Mankiw et al on the other hand presented total ED capital as a ordinary sum of particular kinds of ED capital (se expression ***(4)***).

5.2. On the other hand, if on the top of previous assumptions we now divide second and third part of ***(64)*** with F_K and assume that all relevant parameters are constant, that is $(a_t + b_{ut}) = (a + b_u)$, $(F_{H_0 t} / F_{Kt}) = z_{Ht} = z_H$, and $f_{it} = f_i$, and solve this differential equation, we get

$$Q_t = A_t(z_H H_t + K_t)^{(a+b_u)} \prod_{i=1}^{n} E_{it}^{f_i} \qquad \textbf{(89)}$$

Assuming now only two kind of work, skilled and unskilled, this transform in Griliches like form

$$Q_t = A_t(z_H H_{0t} + z_H H_{1t} + K_t)^{(a+b_u)} E_{1t}^{f_1} = A_t(z_H H_{0t} + z_H H_{1t} + K_t)^{(a+b_u)} (H_{1t} l_{1t})^{f_1} \qquad \textbf{(90)}$$

5.3. In the similar manner and assuming now constancy of ratio of marginal productivity of different kinds of ED capital and marginal productivity of physical capital ($F_{Eit}/F_{Kt} = z_{it}=z_i$), as well as constancy of "raw" labor share in GDP ($b_{ut}=b_u$), constancy of physical share in GDP ($a_t=a$), and constancy of aggregate ED capital share in GDP ($\sum F_{Eit}E_{it}/Q_t=F_{Et}E_t/Q_t=f_t=f$) we can derive following form of production function[30]

$$Q_t = A_t H_t^{b_u} K_t^{a} \left(\sum_{i=1}^{n} z_i E_{it} \right)^{f} = A_t H_t^{b_u} K_t^{a} E_t^{*f}$$

$$= A_t H_t^{b_u} [K_t^{a/(a+f)} E_t^{*f/(a+f)}]^{(a+f)} = A_t H_t^{b_u} C_t^{h} \qquad \textbf{(91)}$$

where

$$E_t^{*} = \sum_{i=1}^{n} z_i E_{it} \qquad \textbf{(92)}$$

and

$$C_t = K_t^{a/(a+f)} E_t^{*f/(a+f)} \qquad \textbf{(93)}$$

Obviously expression ***(91)*** is also very similar to expression ***(2)*** proposed by Mankiw et al (1992). Only difference is in the fact that in expression ***(2)*** ED capital is expressed as ordinary sum of different kinds of ED capital, while here it is expressed as weighted sum of different kinds of ED capital, weights being defined as a ratios of marginal productivity of different kinds of ED capital and marginal productivity physical capital.

5.4. We can with similar assumptions and manipulations as in previous cases derive Griliches like form in this case as well

$$Q_t = A_t[z_H H_t + K_t]^{(a+b_u)} \left(\sum_{i=1}^{n} z_i E_{it} \right)^{f} = A_t[z_H H_{ot} + z_H H_{1t} + K_t]^{(a+b_u)} \left(\sum_{i=1}^{n} z_i E_{it} \right)^{f} \qquad \textbf{(94)}$$

5.5. Let us now take different assumptions about behavior of parameters. First, assume that marginal rate of substitution between different types of ED capital, F_{Ei} / F_{Ej} for any *i* and *j*, does not depend on specific ratio of physical to ED capital, K/E_i for any *i*. Again, it is important because it allows us to solve differential equation ***(64)*** by solving separately each part of those equations. Second assumption is that elasticity of substitution between any two kinds of ED capital is unlimited and independent of quantity of any other kind of ED capital. In other words, changes in ratio of any two kinds of ED capital, E_{it} / E_{jt} for any *i* and *j*, do not have any influence on marginal rate of substitution between those two kinds of inputs, F_{Ei} / F_{Ej} for any *i* and *j*: Corresponding marginal rate of substitution, F_{Ei} / F_{Ej}, is constant. Third, marginal rate of substitution between any kind of ED capital and conventional, physical capital is also assumed to be unlimited: changes in ratio of E_{it} / K_t do not have any influence on marginal rate of substitution between those two kinds of inputs, F_{Ei} / F_K: corresponding marginal rate of substitution is constant. Finally, fifth assumption is that elasticity of substitution between total aggregate capital (conventional and ED capital

[30] Derivation of expression (91) and meaning of underlying assumptions is similar to one given previously for similar expressions.

together) and "raw" labor is equal to one. Consequence is constancy of elasticity of production with respect to "raw" labor and total capital.

If we now for the sake of simplicity mark

$$K_t = E_0 \tag{95}$$

than we can transform equation ***(64)*** in

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + b_{ut}\frac{\dot{H}}{H} + \sum_{i=0}^{n} f_{it}\left(\frac{\dot{E}_i}{E_i}\right) \tag{96}$$

where now

$$f_{0t} = \frac{F_{E_0}E_0}{Q} = \frac{F_K K}{Q} \tag{97}$$

presents elasticity of production with respect to conventional capital. By dividing numerator and denominator of expression ***(96)*** with F_{E0} (that is with F_K) we get

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + b_{ut}\frac{\dot{H}}{H} + h_t\sum_{i=0}^{n}\frac{\left(\frac{F_{E_{it}}}{F_{E_{0t}}}\right)E_{it}}{\sum_{i=0}^{n}\left(\frac{F_{E_{it}}}{F_{E_{0t}}}\right)E_{it}}\left(\frac{\dot{E}_i}{E_i}\right) \tag{98}$$

where new term h_t has following meaning

$$h_t = \sum_{i=0}^{n}\frac{F_{E_i}E_i}{Q} = \frac{\left(F_K K + \sum_{i=1}^{n} F_{E_i}E_i\right)}{Q} = a_t + f_t \tag{99}$$

It presents sum of ED capital and conventional capital share in GDP and it differs from previously given f_t. Having in mind previous assumptions we can write

$$b_{ut} = b_u$$

$$h_t = h \tag{100}$$

$$\frac{F_{E_{it}}}{F_{E_{0t}}} = z_{it} = z_i$$

Substituting this in ***(98)*** we arrive at

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + b_u\frac{\dot{H}}{H} + h\sum_{i=0}^{n}\frac{z_iE_{it}}{\sum_{i=0}^{n} z_iE_{it}}\left(\frac{\dot{E}_i}{E_i}\right) \tag{101}$$

Now, by solving this differential equation (again, by integrating and taking antilogarithm) we get specific production function of the form

$$Q_t = A_tH_t^{b_u}\left(\sum_{i=0}^{n} z_iE_{it}\right)^h = A_tH_t^{b_u}C_t^h = A_tH_t^{b_u}C_t^{(a+f)} \tag{102}$$

where

$$C_t = \sum_{i=0}^{n} z_iE_{it} \tag{103}$$

presents total aggregate capital, conventional and ED together, defined in new way. Total aggregate capital is here combined using arithmetic index. All capital is expressed in efficiency units of conventional capital. Having in mind previously given meaning of E_{0t} (= K_t) and F_{E0} (= F_K) and knowing that $z_0 = F_{E0} / F_{E0} = F_K / F_K = 1$ we can write

$$C_t = z_0 K_t + \sum_{i=1}^{n} z_i E_{it} = K_t + \sum_{i=1}^{n} z_i E_{it} \qquad \textbf{\textit{(104)}}$$

5.6. If we now, on the top of previous assumption, divide with F_K numerator and denominator of second part of expression ***(26)*** and assume that $(F_{H_0 t} / F_{Kt}) = z_{Ht} = z_H$, then solving this differential equation and knowing that $(b_u + h) = 1$, we get linear production function and / or "AK" model of the form

$$Q_t = A_t (b_u + h)\left(z_H H_t + \sum_{i=0}^{n} z_i E_{it} \right) = A_t \left(z_H H_t + K_t + \sum_{i=1}^{n} z_i E_{it} \right) = A_t \overline{K}_t \qquad \textbf{\textit{(105)}}$$

5.7. Let us now compare production function ***(102)*** and particularly this concept of aggregate capital to one proposed by Mankiw (1995) (see expression ***(6)***). General shape of production function ***(102)*** is, having in mind meaning of *h* given in ***(99)***, exactly the same as one given by Mankiw. Aggregate capital is, however, measured in bit a different way. It is here, like in Mankiws function, given as a sum of conventional and ED capital. However, ED capital is now measured in different way: it is given as weighted sum of particular kinds of ED capital, weights being defined as ratio of marginal productivity of that kind of ED capital and marginal productivity of conventional capital. Mankiw's concept of ED capital, on the other hand, is based on assumption that those weights should be equal to one (1) for any ***i*.** In other words, he implicitly assumes that marginal productivity of any kind of ED capital should be equal to marginal productivity of conventional capital, and that rates of return on all kinds of investment are equal. This, as we know, can be true only in economies with perfectly functioning markets.

How realistic is this assumption? At least two sources of difficulties appear here with this assumption. First, process of adjustment to signals from ED capital market is long lasting because of the fact that different vintages of ED capital have long life of at least 40 years and more. Once installed those old vintages of ED capital cannot change so easily and in costless way. New vintages, on the other hand, change educational structure very slowly. So, state of disequilibria and inequality of ED investment returns may least for a longer time. Second, even if this problem disappears, even if market adjustment is instantaneous, what we can get in this case is equality of private rates of return on different kinds of investment. What we need, however, is equality of social rates of return on different investment. Social rates of return (and corresponding marginal productivities) is what count in macroeconomic analysis of this kind. It is well known and documented with different empirical researches that externality of all kinds are widespread in the case of investment in education. So, discrepancy between rates of return in different ED investment can be regarded as rather permanent phenomenon. If this is so, than aggregation of ED capital given in expression ***(104)*** may be regarded as preferable compared to one proposed by Mankiw. Decomposition of rate of growth discussed previously in expressions ***(73)*** and ***(83)*** become in this case meaningful and important indeed. This kind of rate of growth decomposition is not possible with kind of Mankiw production function.

We must admit, however, that for the kind of analysis that tries to reveal sources of differences in level of development of different regions and countries, and this is what Mankiw is trying to do, this approach may be the only one we can afford to use for empirical application. It is, as we know, very difficult and costly to assemble all information necessary for construction of aggregate capital given in ***(104)***. This is especially true in the case of less developed countries. In that case Mankiw's approach is only solution: it is less demanding in data and much easy to apply. On the other hand, in the case of cross-countries analysis, it is not necessarily so problematic, because of the fact that we can quite safely assume that discrepancy between social and private rate of return may have similar shape in all countries.

6. Note at the end that we can in the similar way derive some additional specification of production function with ED capital using previously developed concept of $\boldsymbol{G_t}$ as given in expression ***(82)***

$$\frac{\dot{Q}}{Q}=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\frac{\dot{H}}{H}+\sum_{i=1}^{n}q_{it}\left(\frac{\dot{G}_i}{G_i}\right)=\frac{\dot{A}}{A}+a_t\frac{\dot{K}}{K}+b_{ut}\frac{\dot{H}}{H}+q_t\sum_{i=1}^{n}\left(\frac{F_{G_{it}}G_{it}}{F_{G_t}G_t}\right)\left(\frac{\dot{G}_i}{G_i}\right)$$

If we make certain assumptions, similar to those used to derive expressions in previous paragraphs about movement of respected parameters and solve this differential equations (again by integrating and taking antilogarithm) we can get following production functions.

6.1. First, if we assume constancy of physical capital share (a_t=a), "raw" labor share (b_{ut}=b_u), and particular kind of ED capital share (q_{it}=q_i) in GDP, than we can get following CD production function

$$Q_t=A_tH_t^{b_u}K_t^{a}\prod_{i=1}^{n}G_{it}^{q_i}=A_tH_t^{b_u}\left[K_t^{\frac{a}{a+q}}\prod_{i=1}^{n}G_{it}^{\frac{q_i}{a+q}}\right]^{a+q} \qquad \textbf{(106)}$$

This expression is comparable and similar to previously derived expression ***(85)***. They both suffer from same problem. While assumption of constancy of physical capital share is realistic, assumption of constancy of particular kind of ED capital share in GDP (f_{it}=f_i and q_{it}=q_i) seems to be unrealistic and contradicting to empirical facts.

6.2. If we, however, divide second and third part of expression ***(82)*** with marginal productivity of capital and than assume constancy of relevant parameters, that is $\frac{F_{H0t}}{F_{Kt}}=\chi_{Ht}=\chi_H$, $\frac{F_{Kt}}{F_{Kt}}=1$, q_{it}=q_i, and $(a_t+b_{ut})=(a+b_u)$, than by solving this differential equation we get following Griliches wise function

$$Q_t=A_t[\chi_H H_t+K_t]^{(a+b_u)}\prod_{i=1}^{n}G_{it}^{q_i} \qquad \textbf{(107)}$$

6.3. Next, if we assume constancy of physical capital share (a_t=a), "raw" labor share (b_{ut}=b_u), and total ED capital share (q_t=q) in GDP, and constancy of ratio of marginal productivity of particular kind of ED capital to marginal productivity of physical capital (for $i\in(1, n)$ we will have $\frac{F_{G_{it}}}{F_K}=\chi_{it}=\chi_i$), than we can get following production function

$$Q_t=A_tH_t^{b_u}K_t^{a}\left(\sum_{i=1}^{n}\chi_iG_{it}\right)^{q}=A_tH_t^{b_u}\left[K_t^{a/(a+q)}\left(\sum_{i=1}^{n}\chi_iG_{it}\right)^{q/(a+q)}\right]^{(a+q)} \qquad \textbf{(108)}$$

This expression is analogous to previously derived expression ***(91)***. They are both based on realistic assumptions of constancy of ratio of marginal productivity of particular kind of ED capital to marginal productivity of physical capital ($\frac{F_{E_{it}}}{F_{K_t}} = z_{it} = z_i$ and $\frac{F_{G_{it}}}{F_K} = \chi_{it} = \chi_i$). Therefore, they both seem to be acceptable for analysis of human capital influence on economic development.

6.4. Now, playing with same assumptions as in previous cases we can derive following Griliches kind of function

$$Q_t = A_t[\chi_H H_t + K_t]^{(a+b_u)} \left(\sum_{i=1}^{n} \chi_i G_{it} \right)^q \qquad \textbf{\textit{(109)}}$$

6.5. Next, if we assume constancy of total, physical and aggregate ED, capital share in GDP ($h_t=a_t+q_t=a+q=h$) and "raw" labor share in GDP ($b_{ut}=b_u$), as well as constancy of ratio of marginal productivity of particular kind of capital (ED and physical capital) to marginal productivity of physical capital (for $i \in (0, n)$ we will have $\frac{F_{G_{it}}}{F_{G_{0t}}} = \frac{F_{G_{it}}}{F_K} = \chi_{it} = \chi_i$), than we can get following production function

$$Q_t = A_t H_t^{b_u} \left[\sum_{i=0}^{n} \chi_i G_{it} \right]^{a+q} = A_t H_t^{b_u} \left[K_t + \sum_{i=1}^{n} \chi_i G_{it} \right]^h \qquad \textbf{\textit{(110)}}$$

where $G_{0t}=K_t$, and $\chi_0=1$. This expression is, obviously, comparable to previously derived expression ***(102)***. Same as expressions ***(91)*** and ***(108),*** they are both based on assumptions that are pretty realistic and, therefore, they both seem to be acceptable for analysis of human capital influence on economic development.

6.6. If we divide numerator and denominator of third part of expression ***(82)*** with marginal productivity of physical capital, and on the top of previous assumptions assume that $\frac{F_{Ht}}{F_{Kt}} = \chi_{Ht} = \chi_H$, than, knowing that $a_t+q_t+b_u=1$, we get following linear and / or "AK" production function

$$Q_t = A_t \left[\chi_H H_t + K_t + \sum_{i=1}^{n} \chi_i G_{it} \right] = A_t \overline{K}_t \qquad \textbf{\textit{(111)}}$$

6.7. Finally, if in equation ***(64)*** or ***(82)*** we, first, assume only 3 factors of production (ED capital of skilled labor, unskilled labor, and physical capital), and, second assume constant partial elasticity of substitution between different factors of production, than we can get model formally similar to one proposed by Krusell at all (1997)

$$Q_t = A_t \left(\overline{\mu} H_t^{\overline{\sigma}} + (1-\overline{\mu}) \left(\overline{\lambda} K_t^{\overline{\rho}} + (1-\overline{\lambda}) E_{1t}^{\overline{\rho}} \right)^{\frac{\overline{\sigma}}{\overline{\rho}}} \right)^{\frac{1}{\overline{\sigma}}}$$

$$Q_t = A_t \left(\hat{\mu} H_t^{\hat{\sigma}} + (1-\hat{\mu}) \left(\hat{\lambda} K_t^{\hat{\rho}} + (1-\hat{\lambda}) G_{1t}^{\hat{\rho}} \right)^{\frac{\hat{\sigma}}{\hat{\rho}}} \right)^{\frac{1}{\hat{\sigma}}} \qquad \textbf{\textit{(112)}}$$

4. Concluding remarks

1. In this concluding remarks we will, first, focus on expressions ***(22)***, ***(102)***, ***(6),*** and ***(2)*** from previous considerations. Expression ***(22)*** describes labor input adjusted approach in analysis and measurement of influence of human capital on the level and the rate of economic development. It can be further transformed in the following way[31]

$$Q_t = A_t H_t^{b_u} \left\{ K_t^{\frac{a}{(a+f)}} \left[\sum_{i=1}^{n} n_i H_{it} \left(\sum_{i=1}^{n} \frac{n_i H_{it}}{H_t} \right)^{\frac{b_u}{f}} \right]^{\frac{f}{(a+f)}} \right\}^{(a+f)} = \tag{113}$$

$$A_t H_t^{b_u} \left[K_t^{\frac{a}{(a+f)}} \hat{E}_t^{\frac{f}{(a+f)}} \right]^{(a+f)} = A_t H_t^{b_u} K_t^a \hat{E}_t^f = A_t H_t^{b_u} C_t^{(a+f)}$$

where

$$\hat{E}_t = \sum_{i=1}^{n} n_i H_{it} \left(\sum_{i=1}^{n} \frac{n_i H_{it}}{H_t} \right)^{\frac{b_u}{f}} \tag{114}$$

presents value of educational input / capital expressed in units of marginal productivity of "raw" labor. On the other hand, expression ***(102)*** as we know presents capital adjusted approach that we

[31] Expression (22) can be transformed in

$$Q_t = A_t H_t^{b_u} K_t^a \left(\sum_{i=0}^{n} \frac{n_i H_{it}}{H_t^{b_u/b}} \right)^b = A_t H_t^{b_u} \left[K_t^{\frac{a}{a+f}} \frac{\left(\sum_{i=0}^{n} n_i H_{it} \right)^{\frac{b}{a+f}}}{H_t^{\frac{b_u}{a+f}}} \right]^{(a+f)}$$

Last part in large bracket can be further transformed in

$$K_t^{\frac{a}{a+f}} \left[\left(\sum_{i=0}^{n} n_i H_{it} \right)^b / H_t^{b_u} \right]^{\frac{1}{a+f}} = K_t^{\frac{a}{a+f}} \left[\left(\sum_{i=0}^{n} n_i H_{it} \right)^{b/f} / H_t^{b_u/f} \right]^{\frac{f}{a+f}}$$

Having in mind that

$$b = \sum F_{H_i} H_i / Q = (\sum F_{H_0} H_i + \sum F_{H_i} H_i - \sum F_{H_0} H_i) / Q = [\sum F_{H_0} H_i + \sum (F_{H_i} - F_{H_0}) H_i] / Q =$$
$$\{\sum F_{H_0} H_i + \sum [(F_{H_i} - F_{H_0}) / l_i] H_i l_i\} / Q = \sum F_{H_0} H_i / Q + \sum F_{E_i} E_i / Q = b_u + f$$

previous expression can be transformed in

$$K_t^{\frac{a}{a+f}} \left[\left(\sum_{i=0}^{n} n_i H_{it} \right)^{(b_u+f)/f} / H_t^{b_u/f} \right]^{\frac{f}{a+f}} = K_t^{\frac{a}{a+f}} \left[\sum n_i H_{it} \left(\sum_{i=0}^{n} n_i H_{it} / H_t \right)^{b_u/f} \right]^{\frac{f}{a+f}}$$

Substituting now in above production function we arrive at

$$Q_t = A_t H_t^{b_u} \{K_t^{a/(a+f)} [\sum n_i H_{it} (\sum n_i H_{it} / H_t)^{b_u/f}]^{f/(a+f)}\}^{(a+f)} = A_t H_t^{b_u} \{K_t^{a/(a+f)} \hat{E}^{f/(a+f)}\}^{(a+f)} =$$
$$A_t H_t^{b_u} C_t^{(a+f)} = A_t H_t^{b_u} K_t^a \hat{E}_t^f$$

where obviously

$$C_t = K_t^{a/(a+f)} [\sum n_i H_{it} (\sum n_i H_{it} / H_t)^{b_u/f}]^{f/(a+f)} = K_t^{a/(a+f)} \hat{E}_t^{f/(a+f)}$$

On the other hand

$$\hat{E}_t = \sum n_i H_{it} (\sum n_i H_{it} / H_t)^{b_u/f}$$

It presents value of educational capital expressed in units of marginal productivity of "raw" labor.

developed in previous section assuming heterogeneity of different forms of educational and total capital. Similarly, expression ***(6)*** presents capital adjusted approach, which assume homogeneity of educational capital that has been used in Mankiw (1995) article for same purposes. Finally expression ***(2)*** also present capital adjusted approach, which also assume homogeneity of educational capital, which has been used in Mankiw et al (1992) article.

If we compare those expressions we can notice striking similarities among them: they all look like last line of expression ***(113)***. In all these cases we have Cob Douglas production function with "raw" labor and overall, tangible and intangible, capital, *C*, as inputs. In other words overall capital and "raw" labor are combined like in geometric index using share of overall capital ($a+f$) and share of "raw" labor (b_u) in gross domestic product as weights. As we know share of overall capital is usually somewhere between 2/3 and 3/4, much above the share of conventional capital alone, which is between 1/4 and 1/3. It means that both approaches, capital adjusted and labor adjusted, can be used to express in much stronger way contribution of overall capital to economic development. Differences in productivity, among countries or between different points in time, are in both cases much more sensitive to differences in capital endowment than in classical Solow growth model (see expression ***(1)***). Obviously, capital input adjusted approach proposed one decade ago is not significant novelty in economic analysis. Labor input adjusted approach has been used for same purposes for more than four decades.

However, if we take a look at the meanings of overall capital aggregates, *C*, used in different approaches we will notice important differences among them. In the case of labor input adjusted approach (expression ***(113)***) we have

$$C_t = K_t^{\frac{a}{(a+f)}} \left[\sum_{i=1}^{n} n_i H_{it} \left(\sum_{i=1}^{n} \frac{n_i H_{it}}{H_t} \right)^{\frac{b_u}{f}} \right]^{\frac{f}{(a+f)}} = K_t^{\frac{a}{(a+f)}} \hat{E}_t^{\frac{f}{(a+f)}} \qquad \textbf{\textit{(115)}}$$

In our case of capital adjusted approach with heterogeneous capital, expression ***(102)***, we know that

$$C_t = K_t + \sum_{i=1}^{n} z_i E_{it} \qquad \textbf{\textit{(104)}}$$

In Mankiw (1995) case of capital input adjusted approach that assume homogeneity of educational capital, expression ***(6)***, we have that

$$C_t = K_t + \sum_{i=1}^{n} E_{it} = K_t + E_t \qquad \textbf{\textit{(7)}}$$

Finally, in Mankiw et al (1995) case, expression ***(2)***, we know that

$$C_t = K_t^{\frac{a}{(a+f)}} \left(\sum_{i=1}^{n} E_{it} \right)^{\frac{f}{(a+f)}} = K_t^{\frac{a}{(a+f)}} E_t^{\frac{f}{(a+f)}} \qquad \textbf{\textit{(3)}}$$

In all of these cases overall capital is derived as combination of conventional (tangible) capital and educational (intangible) capital. In the expression ***(115)*** it is geometric combination of conventional capital, *K*, and educational capital, $\hat{E}_t$, with its shares in overall capital income, $a/(a+f)$ and $f/(a+f)$, used as weights. On the other hand, educational capital, $\hat{E}_t$, is here derived as linear combination of different kinds of educational inputs: it is weighted sum of all forms of educational inputs; relative level of productivity of particular kind of educational inputs are here used as weight. In expressions ***(104)*** and ***(7)*** overall capital is derived as a linear combination of conventional and educational capital. It is simple sum of conventional and educational capital. In the first case, expression ***(104)***, educational capital is derived as linear combination of different sort of educational capital; it is weighted sum of all sorts of educational capital; as a weight we use here ratio of particular educational capital marginal productivity to marginal productivity of conventional capital. In other words, overall capital is here presented in efficiency units of conventional capital. In expression ***(7)*** educational capital is simple sum of all sorts of this capital. In fact, expression ***(7)*** assumes that marginal rate of substitution between any kind of educational capital and conventional capital is equal to one. In other words, rate of return is equal for all kinds of investment. In the last case, expression ***(3)***, total capital is presented as a geometric combination of physical and educational capital with a share of physical, $a/(a+f)$, and educational capital, $f/(a+f)$, in total capital income as a weight. Educational capital is here aggregated as ordinary sum of all kind of ED capital ($E_t=\sum E_{it}$).

2. Common characteristic of above four production functions is that they are all based on assumption of unlimited partial elasticity of substitution between different educational inputs. More precisely, in all those cases it is assumed that partial elasticity of substitution between any pars of different educational inputs is unlimited and independent of quantity of any other kind of educational or any other inputs. In other words, changes in ratio of any two kinds of educational inputs do not have any influence on marginal rate of substitution between those two kinds of education: Corresponding marginal rates of substitution are constant. In all of them educational input is, for that reason, aggregated as arithmetic index. In previous two sections we developed some other production functions that are based on same assumption. First, this is case with production functions ***(46)*** and ***(56)*** which are based on heterogeneous labor inputs and which are, therefore, comparable with expression ***(113)***. Next, it applies for production function given by expression ***(110)***. This function is, obviously, comparable to expression ***(102)***. Finally, it also applies for production functions ***(91)*** and ***(108)***. They are similar among themselves and, no doubt, comparable to expressions ***(102)*** and ***(110)***.

In the previous two sections we also developed several other ways of presenting influence of education on level and rate of economic development. First, within second section we developed production functions given by expressions ***(17)***, ***(45)***, and ***(55)***. Second, within third section, we developed production functions given by expressions ***(85)*** and ***(106)***. Common characteristic of those five expressions, and their distinguishing feature compared to above discussed cases, is that they all combine different educational inputs as geometric index. In other world, we are here totally in the realm of Cob Douglass production function. Partial elasticity of substitution between any pars of different kinds of educational inputs is here assumed to be equal to one. Consequently, shares of any kind of ED input in GDP are assumed to be constant. Although very handy, these five production functions have rarely, if ever, been used in empirical researches. Reason for it lays in the fact that their underlying assumptions seem not to mach properly with empirical reality. It is widely known and documented that shares of different kinds of ED capital are not constant. Instead, we have constant increase of share of higher level of ED capital in GDP.

Nevertheless, it is interesting to se what are possible differences among two approach regarding analysis and measurement of influence of human and ED capital on the rate and level of economic development. It is well known that arithmetic index tend to grow faster than geometric index. Consequently, production functions with educational input aggregated using geometric index tend to give smaller importance to human and educational capital to economic development than *analogous* functions that aggregate educational input using arithmetic index. For the same reason, they tend to give greater importance to global factor productivity, A_t. In the case of cross-country analysis, it means that human and ED capital, and in that way capital in general, would have less importance in explaining differences in the level of development between countries than what would be suggested by models of growth used by Mankiw et al (1992) and Mankiw (1995) and with other models suggested here that aggregate human capital and ED input using arithmetic index.

3. So far in empirical researches we have witnessed extensive usage of either approach given by expression ***(22)*** or approach given by expressions ***(6)*** and ***(2)***. Reason for it is in the belief that underlying assumptions of those two approaches mach properly with what we have in reality. Indeed, what we have in reality is relative stability of relative levels of returns to different kinds of investment in education, that is relative stability of rate of returns ratios of any par of different educational capital. Of equal importance is the fact that such stability is supported by theoretical considerations.

Let us now see what are underlying forces that make this assumption so appealing. The answer is simple: those are market forces that tend to equalize private rates of return on all kinds of investment that have same risk premiums. It is very easy to see in the case of homogenous capital adjusted approach given by expression ***(6)***. In fact, as we know, this approach is directly based on the assumption that rates of return on all kinds of education are equal to the rate of return on conventional (tangible) capital. So, assumption of unlimited partial elasticity between different kinds of capital is realistic and acceptable.

However, for equality of rates of return on all kinds of investment to be followed by constancy of F_{Hi} / F_{h0} ratio two additional assumptions have to be fulfilled. First, overall technological progress, that has impact on movement of F_{Hi}, should be unbiased regarding different kinds of education. Second, technological progress within industry of education itself, which has impact on movement of l_i, should be of equal pace in all branches of this sector. These two additional assumptions make labor adjusted approach used by old authors, expression ***(22)***, very restrictive and in that way pretty unrealistic. As we already stressed, economic development of developed countries is in the last three decades characterized with steady increase of wage premium ratio, meaning that two assumptions are not fulfilled. So far several hypothesis have been proposed to explain rising wage premium puzzle. Most promising explanation is one based on idea of capital-skill complementarity. In previous sections we derived several production functions that can be used for that purpose.

4. While it can hardly be denied that market forces tend to equalize private rates of return in different kinds of investment that have same risk premiums, it would equally hardly be to prove that market forces can managed social, or total, rates of return to be equal in all kinds of investment. In fact owing to externalities, positive and negative, we constantly have discrepancies between social and private rates of returns. Investments in education are most notorious example of positive externalities and of those discrepancies. Consequently, what we can expect is higher level of social rates of return on educational capital than that of conventional capital. And we know that what we need in this kind of analysis is relative level of social rate of return, not of private rates of return. In other words, it is much more realistic to assume, as we did in deriving expression ***(102)*** for our heterogeneous capital adjusted model, that ratio of marginal productivity of *i*-th kind of educational capital to marginal productivity of conventional capital ($z_i = F_{Ei} / F_K$) is higher than one and different for different kinds of educational capital. It is what makes our model with heterogeneous educational capital, given in expression ***(102)***, more realistic than that of homogenous capital, given in expression ***(6)***. Being unable to capture externalities, model given in expression ***(6)*** underestimate contribution of educational (and in that way of overall) capital to economic development. In other words, model with heterogeneous educational capital, expression ***(102)***, is more sensitive to rate of investment in overall capital. In that respect it is in accordance with the theoretical models of growth initiated by Robert Lucas (1988, 1993) and Paul Romer

(1986, 1987, 1990, 1994, 1997), which, among other things, put a stress on different forms of externalities as a sources of sustainable growth.

5. Note at the end that our model with heterogeneous capital of education brings one additional benefit. It is able to decompose contribution of education to economic development, either to the rate of growth or to the level of development, in more sophisticated way than either model of growth with labor adjusted input or model of growth with homogenous capital adjusted input. We have seen, discussing expressions ***(73)*** and ***(83)***, that contribution of change in educational structure of labor force, using heterogeneous ED capital approach can be broken up into two parts. First part captures contribution of changes / differences in educational structure of labor force; more precisely it presents contribution of increase of capital of education per capita. Greater increase of capital of education per capita implies stronger improvement of educational structure of labor force, and it implies greater rate of growth. But this improvement and corresponding increase in educational capital per capita can be done in more or less effective way. This is captured with second part, which measures improvement in structure of educational capital. Increase in relative size, $\Delta(E_i / E) > 0$, of those categories of educational capital that have above average relative productivity, $(F_{Eit} / F_{Et}) > 1$, will have positive influence on economic growth, and vice versa. As we suggested earlier it is quite possible that increase of those with basic literacy (followed with decrease of illiterate) be more efficient way of bettering educational structure than increase of university graduate (followed with decrease of those with secondary education). This decomposition is not possible either with labor input adjusted model or with homogenous capital adjusted model.

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