Montenegrin Journal of Economics

Vol. 16, No. 1 (2020), 7-20

# Money and Growth in a MIU-Based Walrasian General Equilibrium Model

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ARTICLE INFO

Received Decmber 07, 2019
Revised from January 10, 2020
Accepted February 21, 2020
Available online March 15, 2020

*JEL classification:* O41; D50

*DOI:* 10.14254/1800-5845/2020.16-1.1

*Keywords:*

Inflation policy,
money,
Walrasian general equilibrium theory,
neoclassical growth theory,
inequality in income and wealth.

ABSTRACT

This paper is concerned with the role of money in economic growth in a general equilibrium framework. It proposes a monetary growth model by integrating Walrasian general equilibrium theory, neoclassical growth theory, and MIU approach in monetary economics with Zhang's concept of disposable income and utility function. We define the model, find equilibrium, and carry out comparative statics analysis in money policy, preferences and technology.

## ACKNOWLEDGEMENTS

I would like to thank the anonymous referee for valuable comments and suggestions. The usual caveat applies.

## INTRODUCTION

The purpose of this study is to formally study relationship between money and economic growth. From empirical as well as theoretical literatures on money and growth, we know that relationships are ambiguous in the sense that the relationship can be positive, independent, or negative not-related, situation-dependent, depending on countries or periods within the same country or analytical frameworks. This study readdresses issues of growth and money. But different from the most formal models in the literature of money and growth, we also study relationship between money and distribution between heterogeneous households. If money is not neutral, inflation policy affects economic growth. As households have various preferences for consumption and saving and varied levels of human capital, it is reasonable to expect that inflation policy should have different effects on income and wealth distribution between heterogenous households. This paper formally shows this intuition. Our modelling framework is based on a few well-known models in economic theory with Zhang's concept of disposable income and utility. They are respectively Walrasian general equilibrium theory, neoclassical growth theory, and monetary growth theory with endogenous capital and money.

In economic theory, modelling money involves intriguing and important questions. As far as this study is concerned, our model is much influenced by Tobin's seminal contribution in 1965 to growth with capital and money. Tobin (1965) introduces money into the Solow model which uses capital accumulation as the economic mechanism of growth (with exogenous technological change and exogeneous population). Withing similarly within an isolated economy like in the Solow model, Tobin introduces "outside money", which is the part of part of money stock issued by the government. Money is competitive with real capital in the portfolios of agents. As in the Solow model, household behavior is not based on utility-maximization. After Tobin published his important model, economists tried to deal with monetary growth problems with microeconomic foundation. One of these approaches is the so-called money in the utility (MIU) function approach. The earlier works are by Patinkin (1965), Sidrauski (1967), and Friedman (1969). The approach assumes that people hold money as it yields some services. Another reason is that it is an easy way to enter real balances directly into the utility function. There is an extensive literature on the approach and empirical studies on monetary issues (e.g., Wang and Yip, 1992, Akinsola, 2017; and Breuer, et al., 2018). This study applies Zhang's MIU approach to determine money holdings by heterogenous households (Zhang, 2005, 2013). Like the Tobin model, this study is still based on neoclassical growth model (e.g., Solow, 1956; Burmeister and Dobell, 1970; and Barro and Sala-i-Martin, 1995). We work in a more general framework than the Solow model. This study follows Uzawa's two sector growth model in describing economic structure and price changes (Uzawa, 1961, 1963; Stiglitz, 1967; Mino, 1996; Drugeon and Venditti, 2001; Jensen, 2003). Rather than a single household, this study classifies the population into different types of households. An extreme case that all households are different as in Walrasian general equilibrium theory (e.g., Walras, 1874; Arrow and Debreu, 1954; Arrow and Hahn, 1971; and Mas-Colell et al., 1995). The theory studies market equilibrium with interdependence between multiple firms and heterogeneous households, production, consumption, and exchanges with heterogeneous industries and households. Our model is Walrasian as the perfect competitive economy has a market equilibrium due to interdependence between profit-maximizing firms and utility-maximizing households for given levels of wealth money. It should be noted that this study is an integration of a growth model with the MIU approach (Zhang, 2013) and a Walrasian growth model (Zhang, 2014). The rest of the study is organized as follows. Section 2 develops the monetary growth model with endogenous wealth and capital with income and wealth distribution between heterogeneous households. Section 3 analyzes properties of the model and identifies the existence of an equilibrium point. Section 4 carries out comparative statics analysis. Section 5 concludes the study.

# 1. THE GROWTH MODEL

We build a Walrasian monetary general equilibrium growth model of endogenous wealth accumulation. The economy produces two goods: capital goods and consumer goods, correspondingly by capital good sector and consumer good sector. We follow the Solow-Uzawa growth model in describing production sectors. The core model in the neoclassical growth theory was the Solow one-sector growth model (Solow, 1956; Uzawa, 1961, 1963; see also Takayama, 1985; Galor, 1992; and Jensen et al., 2001). There are two inputs – labor and capital. Capital depreciates at a constant exponential rate, $\delta_k$. Let $r(t)$ represent the rate of interest. The households hold wealth and money and receive income from wages, and interest payments of wealth. We use Cobb-Douglas production functions to describe technologies of the two sectors.

All markets are perfectly competitive. Inputs are freely mobile between the two sectors. The population is classified into $J$ groups, each group with fixed population, $\bar{N}_j,\ j = 1, \dots, J$. Household $j$ distributes the available time $T_0$ between leisure $\bar{T}_j(t)$ and work $T_j(t)$. Let $N(t)$ for the flow of labor services used for production. We have:

$$N(t) = \sum_{j=1}^{J} h_j \, T_j(t) \, \bar{N}_j, \quad (1)$$

where $h_j$ are the levels of human capital of group $j$.

**The capital good sector.** The capital good sector uses capital and labor as inputs. We use subscript index, $i$ and $s$, to denote respectively the capital good and consumer good sectors. We use $K_m(t)$ and $N_m(t)$ to represent the capital stock and labor force used by sector $m, m = i, s,$ at time $t$. The output level of sector $m$ is given by $F_m(t)$. The capital good sector's production function is taken on the Cobb-Douglas form:

$$F_i(t) = A_i \, K_i^{\alpha_i}(t) \, N_i^{\beta_i}(t), \; \alpha_i, \beta_i > 0, \; \alpha_i + \beta_i = 1, \quad (2)$$

where $A_i, \alpha_i$, and $\beta_i$ are parameters. All the markets are competitive. Input factors are paid according to their marginal products. Firms are faced with the wage rate $w(t)$ and the real rate of interest $r(t)$ determined in markets. The marginal conditions are:

$$r(t) + \delta_k = \frac{\alpha_i \, F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i \, F_i(t)}{N_i(t)}. \quad (3)$$

The wage rate of group $j$ is

$$w_j(t) = h_j \, w(t). \quad (4)$$

**The consumer good sector.** The production function is taken on the Cobb-Douglas form as

$$F_s(t) = A_s \, K_s^{\alpha_s}(t) \, N_s^{\beta_s}(t), \; \alpha_s, \beta_s > 0, \; \alpha_s + \beta_s = 1, \quad (5)$$

where $A_s$, $\alpha_s$ and $\beta_s$ are parameters. We use $p_s(t)$ to stand for the price of consumer good. The marginal conditions imply:

$$r(t) + \delta_k = \frac{\alpha_s \, p_s(t) \, F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s \, p_s(t) \, F_s(t)}{N_s(t)}. \quad (6)$$

**Monetary policy.** Money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money, $M(t) > 0$. The scheme with which the money stock changes over time is known to all agents. The constant net growth rate of money stock is denoted by $\mu$. We thus have:

$$\dot{M}(t) = \mu \, M(t), \; \mu > 0. \quad (7)$$

Let $m(t)$ denote the real value of money per capita measured in units of the output good

$$m(t) = \frac{M(t)}{P(t)}, \quad (8)$$

where $P(t)$ is the price of money. We have the government expenditure per household $\tau(t)$ as follows:

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu\, M(t)}{P(t)} = \mu\, m(t). \quad (9)$$

Each household gets $\mu\, m(t)$ units of paper money from the government. We denote $m_j(t)$ the real money held by household $j$. The money is held by households. We have:

$$\sum_{j=1}^{J} m_j(t)\, \bar{N}_j = m(t)\, \bar{N}, \quad (10)$$

where $\bar{N} \equiv \sum_{j=1}^{J} \bar{N}_j$.

**Current income and disposable income.** Let $\pi(t)$ and $\bar{k}_j(t)$ respectively denote the inflation rate and wealth of household $j$. We have

$$\pi(t) = \frac{\dot{P}(t)}{P(t)}.$$

The current income of household $j$ is as follows:

$$y_j(t) = r(t)\, \bar{k}_j(t) + T_j(t)\, w_j(t) - \pi(t)\, m_j(t) + \tau(t), \quad j = 1, \ldots J, \quad (11)$$

where $r(t)\, \bar{k}_j(t)$ is the interest payment, $w_j(t)$ is the wage income, $\pi(t)\, m_j(t)$ is the real cost of holding money, and $\tau(t)$ is the real value of paper money from the government. The disposable income is given by

$$\hat{y}_j(t) = y_j(t) + a_j(t), \quad (12)$$

where $a_j(t) \equiv \bar{k}_j(t) + m_j(t)$ is the real wealth of household $j$. The total value of wealth that household $j$ can sell to purchase goods and to save is $a_j(t)$. The time constraints imply:

$$T_j(t) + \bar{T}_j(t) = T_0. \quad (13)$$

Insert (11) and (13) in (12)

$$\hat{y}_j(t) = \bar{y}_j(t) - \bar{T}_j(t)\, w_j(t) - (\pi(t) - 1)\, m_j(t), \quad (14)$$

where

$$\bar{y}_j(t) = \big(1 + r(t)\big)\, \bar{k}_j(t) + T_0\, w_j(t) + \tau(t).$$

Household $j$ distributes the disposable income bewteen real money balances $m_j(t)$, saving $s_j(t)$, consumption of goods $c_j(t)$. The budget constraint is given by

$$\big(1 + r(t)\big)\, m_j(t) + c_j(t) + s_j(t) = \hat{y}_j(t). \quad (15)$$

From (14) and (15), we have

$$\bar{T}_j(t)\, w_j(t) + \big(\pi(t) + r(t)\big)\, m_j(t) + p_s(t)\, c_j(t) + s_j(t) = \bar{y}_j(t). \quad (16)$$

The utility level of household $j$ is represented by

$$U_{\square}(t) = \bar{T}_j^{\sigma_{0j}}(t)\, m_j^{\varepsilon_{0j}}(t)\, c_j^{\xi_{0j}}(t)\, s_j^{\lambda_{0j}}(t), \sigma_{0j}, \varepsilon_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

in which $\varepsilon_{0j}$, $\xi_{0j}$, $\sigma_{0j}$, and $\lambda_{0j}$ are called, respectively, propensities to hold money, to consume goods, to use leisure time, and to hold wealth (save). Maximizing $U_j(t)$ subject to (13) yields:

$$w_j(t)\,\bar{T}_j(t) = \sigma_j\,\bar{y}_j(t), \left(\pi(t) + r(t)\right) m_j(t) = \varepsilon_j\,\bar{y}_j(t), p_s(t)\,c_j(t) = \xi_j\,\bar{y}_j(t),$$
$$s_j(t) = \lambda_j\,\bar{y}_j(t), \quad (17)$$

where

$$\sigma_j \equiv \rho_j\,\sigma_{0j}, \varepsilon_j \equiv \rho_j\,\varepsilon_{0j}, \xi_j \equiv \rho_j\,\xi_{0j}, \lambda_j \equiv \rho_j\,\lambda_{0j}, \rho_j \equiv \frac{1}{\sigma_{0j} + \varepsilon_{0j} + \xi_{0j} + \lambda_{0j}}.$$

**Households' wealth accumulation**, According to the definition of $s_j(t)$, the wealth accumulation of household $j's$ wealth change is given by:

$$\dot{a}_j(t) = s_j(t) - a_j(t). \quad (18)$$

**Relations between change in money stock, inflation policy and inflation rate.** According to the definitions and (7), we have:

$$\dot{m}(t) = \frac{\dot{M}(t)}{M(t)} - \frac{\dot{P}(t)}{P(t)} = \left(\mu - \pi(t)\right) m(t). \quad (19)$$

**Full employment of input factors.** Capital stock is fully employed:

$$K_i(t) + K_s(t) = K(t). \quad (20)$$

The total physical wealth is owned by the households:

$$\sum_{j=1}^{J} \bar{k}_j(t)\,\bar{N}_j = K(t). \quad (21)$$

The labor is fully employed:

$$N_i(t) + N_s(t) = N(t). \quad (22)$$

**Market clearing in consumer goods markets.** The demand for consumer goods equals the supply:

$$\sum_{j=1}^{J} c_j(t)\,\bar{N}_j = F_s(t). \quad (23)$$

We constructed the model. The model is structurally a unification of the Walrasian general equilibrium, neoclassical growth theory and MIU approach in monetary economics with Zhang's approach to the household behavior. If wealth accumulation and monetary dynamics are omitted, then the model becomes a Walrasian general equilibrium model. If the population is homogenous and money is omitted, then the model is similar to the Uzawa model in the neoclassical growth theory. If the population is homogeneous, then the model is a monetary neoclassical growth model. Our model is deviated from the mainstreams in economic dynamics in how to model behavior of households. Zhang's approach to household behavior is applied.

## 2. PROPERTIES OF THE ECONOMY

The model is structurally complicated. It includes basic economic mechanisms of Walrasian general economic theory, monetary economics, and neoclassical growth theory. The model is high-dimensional and nonlinear. We now show that the system can be followed with computer. We give a computational procedure to follow the movement of the economy with initial conditions. We introduce a variable:

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

The following lemma is confirmed in the Appendix.

Lemma

The dynamics of the monetary economy with $J$ −group households are described by the following $J + 1$ nonlinear differential equations:

$$\dot{m}(t) = \Omega_0\left(\{\bar{k}_j(t)\}, z(t), m(t)\right),$$
$$\dot{z}(t) = \tilde{\Omega}_1\left(\{\bar{k}_j(t)\}, z(t), m(t)\right),$$
$$\dot{\bar{k}}_j(t) = \tilde{\Omega}_j\left(\{\bar{k}_j(t)\}, z(t), m(t)\right), \ \ j = 2, \dots, J, \ \ (24)$$

where $\Omega_0$ and $\tilde{\Omega}_j$ are functions of $\{\bar{k}_j(t)\} = \left(\bar{k}_2(t), \dots, \bar{k}_J(t)\right)$, $z(t)$, and $m(t)$ defined in the Appendix. The rest variables are given as functions of $\{\bar{k}_j(t)\}$, $z(t)$, and $m(t)$ by the following procedure: $r(t)$ with (A2) → $w(t)$ by (A3) → $w_j(t)$ by (4) → $p_s(t)$ by (A4) → $\bar{k}_1(t)$ by (A15) → $\pi(t)$ by (A16) → $m_j(t)$ by (A17) → $\square_j(t) = \bar{k}_j(t) + m_j(t)$ → $\bar{y}_j(t)$ by (A8) → $\bar{T}_j(t), c_j(t)$, and $s_j(t)$ by (17) → $K(t)$ by (A11) → $K_i(t)$ and $K_s(t)$ by (A6) → $N_i(t)$ and $N_s(t)$ by (A1) → $F_i(t)$ by (2) → $F_s(t)$ by (5) → $m(t)$ by (7) → $P(t)$ by (8) → $Y(t) = F_j(t) + p_s(t) F_s(t)$.

For simulation, we choose $J = 3$ and specify the parameter values:

$$\bar{N}_1 = 50, \bar{N}_2 = 300, \bar{N}_3 = 200, h_1 = 5, h_2 = 3, h_3 = 2, A_i = 1.3, A_s = 1,$$
$$\alpha_i = 0.29, \alpha_s = 0.32, \lambda_{01} = 0.78, \xi_{01} = 0.12, \varepsilon_{01} = 0.02, \sigma_{01} = 0.2,$$
$$\lambda_{02} = 0.75, \xi_{02} = 0.16, \varepsilon_{02} = 0.03, \sigma_{02} = 0.18, \lambda_{03} = 0.7, \xi_{03} = 0.18,$$
$$\varepsilon_{03} = 0.04, \sigma_{01} = 0.15, \mu = 0.03, T_0 = 1, \delta_k = 0.05. \ (25)$$

The three populations are respectively $50, 300,$ and $200$. The levels of human capital are correspondingly $5, 3,$ and $2$. Group $1$ ($3$) has the smallest (largest) population size and highest (lowest) human capital. The inflation policy is fixed at 3 percent. The output elasticities of capital of the capital goods and consumer good sectors are respectively $0.29$ and $0.32$. The three groups have different preferences. As the genuine dynamics is complicated, we are concerned with equilibrium. The system has an equilibrium point as follows:

$$Y = 1560.7, F_i = 743.6, F_s = 646.8, w = 1.19, w_1 = 6, w_2 = 3.58, w_3 = 2.38,$$
$$r = 0.15, m = 1.26, \pi = 0.03, p_s = 1.26, N = 980.5, N_i = 442.7, N_s = 465.9,$$
$$K = 2370.2, K_i = 1071.2, K_s = 1299, \bar{k}_1 = 10.1, \bar{k}_2 = 4.73, \bar{k}_3 = 2.24,$$
$$T_1 = 0.37, T_2 = 0.59, T_3 = 0.71, c_1 = 1.81, c_2 = 1.29, c_3 = 0.84, m_1 = 1.66,$$
$$m_2 = 1.34, m_3 = 1.03, a_1 = 11.73, a_2 = 6.07, a_3 = 3.27, U_1 = 6.59.73,$$
$$U_2 = 3.46, U_3 = 1.85. \ (26)$$

Group 1 has highest wage rate, money holding, physical wealth, wealth, consumption, utility level, group 2's corresponding variables are next, and group 3's corresponding variables are lowest. Household 2 works more than household 1 and less than group 3. The equilibrium inflation rate is equal to the inflation policy.

# 3. COMPARATIVE STATICS ANALYSIS

The previous section provided a computational procedure to calculate the movement of the system and found an equilibrium point. This section carries out comparative statics analysis to examine issues related to money neutrality and effects of changes in preferences and technological changes. This study uses the variable, $\bar{\Delta}x,$ to represent the change rate of the variable $x$ in percentage due to changes in the parameter value.

## 3.1 A rise in inflation policy

We now analyze how the economic equilibrium is affected when the inflation policy is increased as follows: $\mu: 0.03 \Rightarrow 0.035,$ where "⇒" stands for "being changed to". The result is listed in (27). We see that money is not neutral in the long term. A rise in inflation increases cost of holding money. All the households hold less (real) money and have more wealth. As inflation is increased, the households tend to hold more real wealth $\bar{k}_j$. The national real money is decreased. The total physical capital stock is increased. Each sector employs more capital. The wage rates are increased. The households work less hours as net results of rises in the wage rates and wealth. The national labor supply is reduced. They consume more consumer goods and have higher levels of utility. The rate of interest falls. The capital good sector produces less. The consumer good sector produces more in association with falling price.

$$\bar{\Delta}Y = 0.17, \bar{\Delta}F_i = -0.68, \bar{\Delta}F_s = 0.98, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = 0.47,$$
$$\bar{\Delta}r = -1.53, \bar{\Delta}m = -0.48, \bar{\Delta}\pi = 16.7, \bar{\Delta}p_s = -0.05, \bar{\Delta}N = -0.32, \bar{\Delta}N_i = -1.14,$$
$$\bar{\Delta}N_s = 0.46, \bar{\Delta}K = 1.37, \bar{\Delta}K_i = 0.48, \bar{\Delta}K_s = 2.11, \bar{\Delta}\bar{k}_1 = 0.89, \bar{\Delta}\bar{k}_2 = 1.38,$$
$$\bar{\Delta}\bar{k}_3 = 1.9, \bar{\Delta}T_1 = -0.3, \bar{\Delta}T_2 = -0.33, \bar{\Delta}T_3 = -0.31, \bar{\Delta}c_1 = 0.65, \bar{\Delta}c_2 = 0.96,$$
$$\bar{\Delta}c_3 = 1.22, \bar{\Delta}m_1 = -0.82, \bar{\Delta}m_2 = -0.52, \bar{\Delta}m_3 = -0.26, \bar{\Delta}a_1 = 0.65,$$
$$\bar{\Delta}a_2 = 0.96, \bar{\Delta}a_3 = 1.22, \bar{\Delta}U_1 = 0.61, \bar{\Delta}U_2 = 0.94, \bar{\Delta}U_3 = 1.18. \quad (27)$$

Our results re similar to those obtained through the Tobin model (Tobin, 1956) which is a monetary neoclassical growth model identical to the Solow model when inflation policy is zero. Although the Tobin model is not based on utility-maximization like our model, demand and supply are similar to our model. The Tobin model predicts that the long-term capital and consumption are positively related to inflation policy. In Tobin's approach, money and capital are substitutes. Portfolio choice between money and capital is affected by inflation policy. A higher inflation reduces the rate of return of money, which would increase capital in the economy. More capital enhances output. Hence consumption will be increased. It should be noted that in the literature of monetary economic growth, different approaches provide opposite relationships between money and growth. Sidrauski (1967) applied the Ramsey approach and got the superneutrality of money. Stockman (1981) introduced money in the expenditure function and got the anti-Tobin effects.

## 3.2 Household 1 increases the propensity to hold money

We now analyze how the economic equilibrium is affected when household 1 increases the propensity to hold money as follows: $\varepsilon_{01}: 0.02 \Rightarrow 0.023.$ The result is listed in (28). Household 1 has more money, while each household in the other two groups has less money. Household 1 has more money, while each household in the other two groups has less money. Household 1 holds less physical capital and wealth, consumes less consumer goods, and works more hours, while each household from the other two groups holds more physical capital and wealth, consumes more consumer goods, and works less hours. Household 1's utility is lowered, while each household from the other two groups has higher utility. The wage rate is reduced in association in rises in the rate of interest. The national money is increased. The total labor supply is enhanced, but national physical capital is reduced. Each sector employs more labor but less capital. The national output is slightly augmented. The capital good sector produces more, while the consumer good sector produces less in association with rises in the price.

$$\bar{\Delta}Y = 0.004, \bar{\Delta}F_i = 0.42, \bar{\Delta}F_s = -0.41, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = -0.4,$$
$$\bar{\Delta}r = 1.3, \bar{\Delta}m = 0.32, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = 0.04, \bar{\Delta}N = 0.41, \bar{\Delta}N_i = 0.82, \bar{\Delta}N_s = 0.03,$$
$$\bar{\Delta}K = -0.98, \bar{\Delta}K_i = -0.55, \bar{\Delta}K_s = -1.33, \bar{\Delta}\bar{k}_1 = -7.12, \bar{\Delta}\bar{k}_2 = 0.63, \bar{\Delta}\bar{k}_3 = 0.82,$$
$$\bar{\Delta}T_1 = 7.83, \bar{\Delta}T_2 = -0.5, \bar{\Delta}T_3 = -0.3, \bar{\Delta}c_1 = -4.97, \bar{\Delta}c_2 = 0.33, \bar{\Delta}c_3 = 0.33,$$
$$\bar{\Delta}m_1 = 8.12, \bar{\Delta}m_2 = -0.75, \bar{\Delta}m_3 = -0.75, \bar{\Delta}a_1 = -4.97, \bar{\Delta}a_2 = 0.33,$$
$$\bar{\Delta}a_3 = 0.33, \bar{\Delta}U_1 = -5.28, \bar{\Delta}U_2 = 0.41, \bar{\Delta}U_3 = 0.36. \quad (28)$$

## 3.3 Household 3's human capital is enhanced

We now analyze how the economic equilibrium is affected when household 3's human capital is enhanced as follows: $h_3: 2 \Rightarrow 2.2.$ The result is listed in (29). Household 3's wage rate is enhanced, while the other two groups' wage rates are reduced. Household 3 has more money, owns more physical capital, has more wealth, while each household from the other groups has less money, but owns more physical capital and has more wealth. All the households consume more consumer goods, enjoy more leisure hours, and have higher utilities. The national money is enhanced and rate of interest is increased. The total labor supply and national physical capital are augmented. Each sector employs more labor but less labor force. The national output is augmented. The two sectors both produce more. The price of consumer goods rises slightly.

$$\bar{\Delta}Y = 2.67, \bar{\Delta}F_i = 2.56, \bar{\Delta}F_s = 2.74, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = -0.19, \bar{\Delta}w_3 = 9.79,$$
$$\bar{\Delta}r = 0.62, \bar{\Delta}m = 2.59, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = 0.02, \bar{\Delta}N = 2.86, \bar{\Delta}N_i = 2.76, \bar{\Delta}N_s = 2.96,$$
$$\bar{\Delta}K = 2.2, \bar{\Delta}K_i = 2.09, \bar{\Delta}K_s = 2.29, \bar{\Delta}\bar{k}_1 = 0.24, \bar{\Delta}\bar{k}_2 = 0.33, \bar{\Delta}\bar{k}_3 = 10.31,$$
$$\bar{\Delta}T_1 = -0.58, \bar{\Delta}T_2 = -0.26, \bar{\Delta}T_3 = -0.1, \bar{\Delta}c_1 = 0.15, \bar{\Delta}c_2 = 0.18, \bar{\Delta}c_3 = 10.05,$$
$$\bar{\Delta}m_1 = -0.37, \bar{\Delta}m_2 = -0.33, \bar{\Delta}m_3 = 9.49, \bar{\Delta}a_1 = 0.15, \bar{\Delta}a_2 = 0.18, \bar{\Delta}a_3 = 10.05,$$
$$\bar{\Delta}U_1 = 0.21, \bar{\Delta}U_2 = 0.22, \bar{\Delta}U_3 = 9.23. \quad (29)$$

## 3.4 Group 3's population is enlarged

We now analyze how the economic equilibrium is affected when group 3's population is enlarged as follows: $\bar{N}_3: 2 \Rightarrow 2.2.$ The result is listed in (30). The wage rate is reduced, and the rate of interest is enhanced. Each household holds less money, owns more physical capital and has more wealth. All the households consume more, work less, and enjoy higher levels of utility. The national money per household is reduced and rate of interest is increased. The total labor supply and national physical capital are augmented. Each sector employs more labor and capital. The national output is augmented. The two sectors both produce more. The price of consumer goods rises slightly.

$$\bar{\Delta}Y = 1.34, \bar{\Delta}F_i = 1.28, \bar{\Delta}F_s = 1.37, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = -0.1,$$
$$\bar{\Delta}r = 0.32, \bar{\Delta}m = -0.51, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = 0.01, \bar{\Delta}N = 1.43, \bar{\Delta}N_i = 1.38,$$
$$\bar{\Delta}N_s = 1.48, \bar{\Delta}K = 1.1, \bar{\Delta}K_i = 1.04, \bar{\Delta}K_s = 1.14, \bar{\Delta}\bar{k}_1 = 0.11, \bar{\Delta}\bar{k}_2 = 01.5,$$
$$\bar{\Delta}\bar{k}_3 = 01.9, \bar{\Delta}T_1 = -0.28, \bar{\Delta}T_2 = -0.12, \bar{\Delta}T_3 = -0.07, \bar{\Delta}c_1 = 0.07, \bar{\Delta}c_2 = 0.07,$$
$$\bar{\Delta}c_3 = 0.07, \bar{\Delta}m_1 = -0.2, \bar{\Delta}m_2 = -0.19, \bar{\Delta}m_3 = -0.19, \bar{\Delta}a_1 = 0.07,$$
$$\bar{\Delta}a_2 = 0.07, \bar{\Delta}a_3 = 0.07, \bar{\Delta}U_1 = 0.1, \bar{\Delta}U_2 = 0.09, \bar{\Delta}U_3 = 0.08. \quad (30)$$

## 3.5 Household 3's propensity to save is increased

We now analyze how the economic equilibrium is affected when household 3's propensity to save is enhanced as follows: $\lambda_{03}: 0.7 \Rightarrow 0.71$. The result is listed in (31). Household 3 has more wealth and own more physical capital, while each household from the other groups has less wealth and own less physical capital. All the households hold more money. Household 3 has more leisure time and consume more goods, while each household from the other groups has less leisure time and consume less. Household 3 enjoys higher utility, while each household from the other groups has lower utility. The national money is enhanced, and rate of interest is reduced. The total labor supply and national physical capital are augmented. Each sector employs more labor and capital. The national output is augmented. The two sectors both produce more. The price of consumer goods falls slightly.

$$\bar{\Delta}Y = 0.14, \bar{\Delta}F_i = 0.28, \bar{\Delta}F_s = 0.02, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = 0.08,$$
$$\bar{\Delta}r = -0,26, \bar{\Delta}m = 0.24, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = -0.01, \bar{\Delta}N = 0.06, \bar{\Delta}N_i = 0.2,$$
$$\bar{\Delta}N_s = -0.07, \bar{\Delta}K = 0.33, \bar{\Delta}K_i = 0.48, \bar{\Delta}K_s = 0.2, \bar{\Delta}\bar{k}_1 = -0.09, \bar{\Delta}\bar{k}_2 = -0.12,$$
$$\bar{\Delta}\bar{k}_3 = 2.22, \bar{\Delta}T_1 = 0.23, \bar{\Delta}T_2 = 0.1, \bar{\Delta}T_3 = -0.06, \bar{\Delta}c_1 = -0.05, \bar{\Delta}c_2 = -0.06,$$
$$\bar{\Delta}c_3 = 0.23, \bar{\Delta}m_1 = 0.16, \bar{\Delta}m_2 = 0.15, \bar{\Delta}m_3 = 0.45, \bar{\Delta}a_1 = -0.05, \bar{\Delta}a_2 = -0.06,$$
$$\bar{\Delta}a_3 = 1.66, \bar{\Delta}U_1 = -0.08, \bar{\Delta}U_2 = -0.08, \bar{\Delta}U_3 = 2.47. \quad (31)$$

## 3.6 Household 3's propensity to use leisure time is increased

We now analyze how the economic equilibrium is affected when household 3's propensity to use leisure time is enhanced as follows: $\sigma_{03}: 0.15 \Rightarrow 0.16$. The result is listed in (32). Household 3 has more leisure time, while each household from the other groups work more. Household 3 holds less money, while each household from the other groups holds more. All the households have less wealth and less physical capital. The wage rate is slightly increased. All the households have lower utility levels. The national money is reduced. The rate of interest is reduced. The total labor supply and national physical capital are reduced. Each sector employs less labor and capital. The national output is reduced. The two sectors both produce less. The price of consumer goods falls slightly.

$$\bar{\Delta}Y = -0.51, \bar{\Delta}F_i = -0.49, \bar{\Delta}F_s = -0.53, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = 0.04,$$
$$\bar{\Delta}r = -0.12, \bar{\Delta}m = -0.5, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = -0.004, \bar{\Delta}N = -0.55, \bar{\Delta}N_i = -0.53,$$
$$\bar{\Delta}N_s = -0.57, \bar{\Delta}K = -0.42, \bar{\Delta}K_i = -0.4, \bar{\Delta}K_s = -0.44, \bar{\Delta}\bar{k}_1 = -0.05,$$
$$\bar{\Delta}\bar{k}_2 = -0.07, \bar{\Delta}\bar{k}_3 = -1.98, \bar{\Delta}T_1 = 0.11, \bar{\Delta}T_2 = 0.05, \bar{\Delta}T_3 = -1.9, \bar{\Delta}c_1 = -0.03,$$
$$\bar{\Delta}c_2 = -0.04, \bar{\Delta}c_3 = -1.93, \bar{\Delta}m_1 = 0.07, \bar{\Delta}m_2 = 0.07, \bar{\Delta}m_3 = -1.83, \bar{\Delta}a_1 = -0.03,$$
$$\bar{\Delta}a_2 = -0.04, \bar{\Delta}a_3 = -1.93, \bar{\Delta}U_1 = -0.04, \bar{\Delta}U_2 = -0.04, \bar{\Delta}U_3 = -2.27. \quad (32)$$

## 3.7 The consumer good sector's total factor productivity is enhanced

We now analyze how the economic equilibrium is affected when the consumer good sector's total factor productivity is enhanced as follows: $A_s: 1 \Rightarrow 1.05.$ The result is listed in (33). All the households hold more money and have less wealth and less physical capital. They all work more hours and consume less consumer goods. Their utility levels are lowered. The wage rate is slightly increased. The national money is augmented. The rate of interest is reduced. The total labor supply is increased, while the national physical capital is reduced. The capital good sector employs more inputs, while the consumer good sector employs less inputs. The capital good sector produces more, while the consumer good sector produces less. The price of consumer goods falls.

$$\bar{\Delta}Y = 0.02, \bar{\Delta}F_i = 5.33, \bar{\Delta}F_s = -0.04, \bar{\Delta}w = \bar{\Delta}w_1 = \bar{\Delta}w_2 = \bar{\Delta}w_3 = 0.06,$$
$$\bar{\Delta}r = -0.19, \bar{\Delta}m = 0.11, \bar{\Delta}\pi = 0, \bar{\Delta}p_s = -4.77, \bar{\Delta}N = 0.07, \bar{\Delta}N_i = 5.27,$$
$$\bar{\Delta}N_s = -4.86, \bar{\Delta}K = -0.09, \bar{\Delta}K_i = 5.47, \bar{\Delta}K_s = -4.68, \bar{\Delta}\bar{k}_1 = -0.06,$$
$$\bar{\Delta}\bar{k}_2 = -0.09, \bar{\Delta}\bar{k}_3 = -0.12, \bar{\Delta}T_1 = 0.16, \bar{\Delta}T_2 = 0.07, \bar{\Delta}T_3 = 0.04, \bar{\Delta}c_1 = -0.04,$$
$$\bar{\Delta}c_2 = -0.05, \bar{\Delta}c_3 = -0.04, \bar{\Delta}m_1 = 0.12, \bar{\Delta}m_2 = 0.11, \bar{\Delta}m_3 = 0.11, \bar{\Delta}a_1 = -0.04,$$
$$\bar{\Delta}a_2 = -0.05, \bar{\Delta}a_3 = -0.04, \bar{\Delta}U_1 = -0.06, \bar{\Delta}U_2 = -0.06, \bar{\Delta}U_3 = -0.05. \quad (33)$$

## CONCLUSIONS

This paper proposed a dynamic equilibrium growth model to examine the relationship between economic growth and money growth by integrating the Walrasian general equilibrium theory, neoclassical growth theory, and MIU approach in monetary economics with the concept of disposable income and utility function. We defined the model, found equilibrium, and carried out comparative statics analysis in money policy, preferences and technology. The paper deals with a complicated topic and is involved different ideas in economic theory. It is built on some strict assumptions.

Although this paper is theoretical one and is not empirically tested. It might provide some insights into complexity of monetary economic growth. Traditional theories which have been empirically tested based don't give any convergent conclusions on the role of money in economic growth. Our theoretical model which is based on multiple forces of economic growth well accepted in the literature of economic growth provides some insights into situation-dependent conclusions in the literature of empirical studies on the role of government's monetary policy. The paper can be extended in different ways. It is important to use more general forms of production functions and utility functions. The government's monetary policy is oversimplified. Growth money is an endogenous variable of social and economic changes. For instance, we may replace the fixed money growth rate policy with the Taylor rule. Issues related to government debts are a main concern in the literature. It is conceptually not difficult to examine interdependence between money growth and national debts on the basis of our model.

### Appendix: Confirming the Lemma

With (3) and (6), we get:

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_s N_s}{K_s}, \quad (A1)$$

where time index is omitted and $\tilde{\alpha}_j \equiv \alpha_j/\beta_j, j = i, s.$ From (2) and (3), we get

$$r(z) = \frac{\alpha_i A_i z^{\beta_i}}{\tilde{\alpha}_i^{\beta_i}} - \delta_k. \quad (A2)$$

where we also use (A1). Equations (A1) also imply:

$$w(z) = \frac{r + \delta_k}{z}. \ (A3)$$

From (5), (6) and (A1), we have

$$p_s(z) = \frac{z^{\alpha_s} w}{A_s \beta_s \tilde{\alpha}_s^{\alpha_s}}. \ (A4)$$

Insert (A1) in (22)

$$\left(\frac{K_i}{\tilde{\alpha}_i} + \frac{K_s}{\tilde{\alpha}_s}\right) z = N. \ (A5)$$

Solve (A5) and (20)

$$K_i = \frac{\tilde{\alpha} N}{z} - \frac{\tilde{\alpha} K}{\tilde{\alpha}_s}, K_s = \frac{\tilde{\alpha} K}{\tilde{\alpha}_i} - \frac{\tilde{\alpha} N}{z}, \ (A6)$$

where

$$\tilde{\alpha} \equiv \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_s}\right)^{-1}.$$

By (A6) and (A1), we have

$$N_i = \left(N - \frac{z K}{\tilde{\alpha}_s}\right) \frac{\tilde{\alpha}}{\tilde{\alpha}_i}, N_s = \left(\frac{z K}{\tilde{\alpha}_i} - N\right) \frac{\tilde{\alpha}}{\tilde{\alpha}_s}. \ (A7)$$

From (14), (9) and (4), we have

$$\bar{y}_j = R \bar{k}_j + T_0 h_j w + \mu m. \ (A8)$$

where $R \equiv 1 + r$. Insert (17) in (A8)

$$\bar{T}_j = \frac{R \sigma_j \bar{k}_j}{w_j} + \sigma_j T_0 + \frac{\sigma_j \mu m}{w_j}. \ (A9)$$

By (A9) and (13), we have

$$T_j = (1 - \sigma_j) T_0 - \frac{R \sigma_j \bar{k}_j}{w_j} - \frac{\sigma_j \mu m}{w_j}. \ (A10)$$

From (A10) and (1), we have

$$N = \sigma_0 - \bar{\sigma} m - \sum_{j=1}^{J} \bar{\sigma}_j \bar{k}_j, \ (A11)$$

where

$$\sigma_0 \equiv T_0 \sum_{j=1}^{J} (1 - \sigma_j) h_j \bar{N}_j, \ \bar{\sigma}_j \equiv \frac{R \sigma_j \bar{N}_j}{w}, \ \bar{\sigma} \equiv \frac{\mu}{w} \sum_{j=1}^{J} \sigma_j \bar{N}_j.$$

Insert (17) in (23):

$$\sum_{j=1}^{J} \xi_j \bar{N}_j \bar{y}_j = \frac{w N_s}{\beta_s}, \quad (A12)$$

where we also use (6). Insert (A8) and (A7) in (A12)

$$\sum_{j=1}^{J} \bar{\xi}_j \bar{k}_j + h + \bar{\xi} m = \frac{z K}{\tilde{\alpha}_i} - N. \quad (A13)$$

where

$$\bar{\xi}_j \equiv \frac{\alpha_s R \xi_j \bar{N}_j}{\tilde{\alpha} w}, \quad h \equiv \frac{\alpha_s T_0}{\tilde{\alpha}} \sum_{j=1}^{J} \xi_j \bar{N}_j h_j, \quad \bar{\xi} \equiv \frac{\alpha_s \mu m}{\tilde{\alpha} w} \sum_{j=1}^{J} \xi_j \bar{N}_j .$$

Substituting (A11) and (21) into (A13) yields

$$\sum_{j=1}^{J} \left( \bar{\xi}_j - \frac{z \bar{N}_j}{\tilde{\alpha}_i} - \bar{\sigma}_j \right) \bar{k}_j + h + \sigma_0 + \left( \bar{\xi} - \bar{\sigma} \right) m = 0. \quad (A14)$$

Solving (A14) with $\bar{k}_1$ as the variable, we have

$$\bar{k}_1 = \Lambda(\{\bar{k}_j\}, z, m) \equiv$$
$$\left( \sum_{j=2}^{J} \left( \bar{\xi}_j - \frac{z \bar{N}_j}{\tilde{\alpha}_i} - \bar{\sigma}_j \right) \bar{k}_j + h + \sigma_0 + \left( \bar{\xi} - \bar{\sigma} \right) m \right) \left( \frac{z \bar{N}_1}{\tilde{\alpha}_i} + \bar{\sigma}_1 - \bar{\xi}_1 \right)^{-1} . \quad (A15)$$

Insert (A8) in (A14)

$$\pi = \tilde{\Lambda}(\{\bar{k}_j\}, z, m) \equiv \frac{R}{m \bar{N}} \sum_{j=2}^{J} \varepsilon_j \bar{N}_j \bar{k}_j + \bar{r}, \quad (A16)$$

where

$$\bar{r}(z) \equiv \frac{w T_0}{m \bar{N}} \sum_{j=1}^{J} \varepsilon_j h_j \bar{N}_j + \frac{\mu}{\bar{N}} \sum_{j=1}^{J} \varepsilon_j \bar{N}_j - r.$$

By (17), (A15) and (A16), we have:

$$m_j = \Lambda_j(\{\bar{k}_j\}, z, m) \equiv \frac{\varepsilon_j \left( R \bar{k}_j + T_0 h_j w + \mu m \right)}{\pi + r}, \quad (A17)$$

where we also use (A14). It is straightforward to check that all the variables can be expressed as functions of $\{\bar{k}_j\}$, $z$, and $m$ at any point in time as follows: $r$ with (A2) $\rightarrow w$ by (A3) $\rightarrow w_j$ by (4) $\rightarrow p_s$ by (A4) $\rightarrow \bar{k}_1$ by (A15) $\rightarrow \pi$ by (A16) $\rightarrow m_j$ by (A17) $\rightarrow a_j = \bar{k}_j + m_j \rightarrow \bar{y}_j$ by (A8) $\rightarrow \bar{T}_j$ $c_j$, and $s_j$ by (17) $\rightarrow K$ by (A11) $\rightarrow K$ by (21) $\rightarrow K_i$ and $K_s$ by (A6) $\rightarrow N_i$ and $N_s$ by (A1) $\rightarrow F_i$ by (2) $\rightarrow F_s$ by (5) $\rightarrow m$ by (7) $\rightarrow P$ by (8) $\rightarrow Y = F_j + p_s F_s$. From this procedure, (18) and (19), we get:

$$\dot{\bar{k}}_j + \dot{m}_j = \Omega_j\left(\{\bar{k}_j\}, z, m\right) \equiv s_j - a_j, \quad (A18)$$
$$\dot{m} = \Omega_0\left(\{\bar{k}_j\}, z, m\right) \equiv (\mu - \pi)\, m. \quad (A19)$$

Take derivative of (A17) in time

$$\dot{m}_j = \sum_{j=2}^{J} \frac{\partial \Lambda_j}{\partial \bar{k}_q} \dot{\bar{k}}_q + \frac{\partial \Lambda_j}{\partial z} \dot{z} + \Omega_0 \frac{\partial \Lambda_j}{\partial m}. \quad (A20)$$

in which we use (A18) and (A19). Similarly, from (A15) we get

$$\dot{\bar{k}}_1 = \sum_{j=2}^{J} \frac{\partial \Lambda}{\partial \bar{k}_j} \dot{\bar{k}}_j + \frac{\partial \Lambda}{\partial z} \dot{z} + \Omega_0 \frac{\partial \Lambda}{\partial m}. \quad (A21)$$

Insert (A20) and (A21) in (A18)

$$\sum_{q=2}^{J} \left(\frac{\partial \Lambda}{\partial \bar{k}_q} + \frac{\partial \Lambda_1}{\partial \bar{k}_q}\right) \dot{\bar{k}}_q + \left(\frac{\partial \Lambda}{\partial z} + \frac{\partial \Lambda_1}{\partial z}\right) \dot{z} = \Omega_1 - \Omega_0 \frac{\partial \Lambda}{\partial m} - \Omega_0 \frac{\partial \Lambda_1}{\partial m},$$
$$\dot{\bar{k}}_j + \sum_{j=2}^{J} \frac{\partial \Lambda_j}{\partial \bar{k}_q} \dot{\bar{k}}_q + \frac{\partial \Lambda_j}{\partial z} \dot{z} = \Omega_j - \Omega_0 \frac{\partial \Lambda_j}{\partial m}. \quad (A22)$$

Equations (A22) are linear in $\left\{\dot{\bar{k}}_j\right\}$ and $\dot{z}$. It is straightforward to solve the linear equations. As it is tedious to give the explicit expressions, we express the solution of (A22) in the form of differential equations as follows:

$$\dot{z} = \tilde{\Omega}_1\left(\{\bar{k}_j\}, z, m\right),$$
$$\dot{\bar{k}}_j = \tilde{\Omega}_j\left(\{\bar{k}_j\}, z, m\right), \quad j = 2, \dots, J. \quad (A23)$$

Differential equations (A19) and $(A23)$ have $J + 1$ equations and contain the same number of variables $\{\bar{k}_j\}$, $z$, and $m$. We thus confirm the Lemma.

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